Data Security Threat Evaluation Using Bayesian Prioritization Method

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***Abstract*- Accelerating complexity of threat management requires the use of more flexible approaches to measure data security threat. Adapting convoluted threat analysis tools in today's data system is a very tough task due to the shortage of reliable data. Analytic Hierarchy Process group decision making (AHP-GDM) offers a backing for threat analysis by taking the judgements of managers and systematically computing the relative threat values. This paper presents how Bayesian Prioritization procedure (BPP) provides a more productive way of threat evaluation than proposed by the conventional approaches used in AHP-GDM.**

***Keywords*: Data security, Threat evaluation, Analytic hierarchy process (AHP), Group decision making (GDM), Bayesian prioritization procedure (BPP).**

1.INTRODUCTION

Data security threat management is a recurrent process of identification, evaluation and prioritization of threats, where threat could be defined as a possibility that a threat exploits a particular vulnerability in an asset and causes damage or loss to the asset. Threat management has two primary activities, threat evaluation and threat control.Threat evaluation is a very important decision mechanism which identifies the data security assets that are vulnerable to threats, calculates the quantitative or qualitative value of threat (or expected loss), and prioritizes threat incidents. In an organization, in the past, a single manager was used to be the responsible staff to protect data systems where, nowadays, a group of managers could take the responsibility of this task or participate in the threat assessment process. As threat analysis becomes a cross-functional decision making process, researchers seek ways to develop new threat analysis methods which allow a group of people to participate.

Although threat is well defined and practical for decision making, it is often difficult to calculate a priori [1]. Due to the difficulty in incorporating abstruse threat analysis tools in today's data systems, researchers have proposed new techniques which are capable of analyzing data security threat properly. A number of quantitative and qualitative threat analysis methods have been developed.

The quantitative approaches use mathematical and statistical tools to represent threat as a function of the probability of a threat and the expected loss due to the vulnerability of the organization to this threat [2,3]. Due to the shortage of dependable data on incidents (probabilities and impacts), quantitative approaches may not yield trustworthy results. Consequently, security or threat management professionals mostly prefer qualitative methods rather than quantitative ones. In qualitative methods, estimated threat is calculated using only the estimated potential loss instead of the probability data. These approaches rely on the ideas of the analyst so they are subjective and might yield inconsistent results[4]. There is not a single threat evaluation method which is best under all circumstances and for all purposes. Some researchers claimed that neither of the quantitative and qualitative approaches could properly model the evaluation process alone. Alternatively, some of them developed comprehensive approaches combining both the quantitative and the qualitative approaches [2,3,5]. The Analytic Hierarchy Process (AHP), first proposed by T. L. Saaty [6], is one of the most widely used multi-criteria decision technique which can combine qualitative and quantitative factors for prioritizing, ranking and evaluating alternatives [7]. It allows multiple actors, criteria and scenarios to be involved in the analysis [8].

Previously, AHP analysis was used as support for an organization's data security system to evaluate the weights of threat factors [9], to determine the optimal allocation of a budget [10], to evaluate the weighting factors needed to combine threat measures [2], to obtain the indices' weights with respect to the final goal of the security evaluation [11], to select data security policy [12], and to establish e-commerce data security evaluation [13]. Zhang et al. [14] proposed calculating a relative threat value with Analytic Hierarchy Process group decision making (AHP-GDM) instead of calculating the actual value of the threat. They mentioned that the loss could be measured by the value of assets, and that possibility of threat could be described in an equation with the danger level of threat and vulnerability as its two variables.

The AHP method is operable and efficient as it prioritizes and orders threat incidents, which could also satisfy the aim of threat management. However, there might be some complexities when using AHP-GDM for data security threat evaluation. For instance, in AHP-GDM, it is assumed that the pairwise comparison matrices containing the judgements expressed by decision makers are complete and precise. In real life, decision makers might provide only incomplete data due to following situations: (1) some of the decision makers may have restricted expertise about the problem domain or the AHP analysis; (2) decision makers participated in the analysis would prefer to concentrate on the threat evaluation itself rather than the AHP tool being implemented in the threat analysis; (3) they may have difficulties in making pairwise comparisons efficiently as the number of elements (assets, threats and vulnerabilities) in the problem mount. Moreover, the practitioner may also prefer to ignore the inconsistent or repelling judgements while keeping the consistent or homogeneous ones in order to increase the consistency or accord among decision makers. Altuzarra et al. [15] proposed a Bayesian prioritization approach for AHP-GDM which can naturally be extended to the case of incomplete pairwise comparison matrices. Contrary to the conventional prioritization methods applied in AHP-GDM [16-18], this technique does not require intermediate filters for decision makers' initial judgements.

The paper aims at providing an effective and practical group decision mechanism to prioritize the threat incidents. We propose using BPP based AHP-GDM for data security threat evaluation, which is a cure for the convolutions mentioned above. This approach provides flexibility to the group of participants when expressing their judgements, and to the threat analysts, who may be novice AHP practitioners, by treating incomplete or inconsistent judgements properly. We compare the method with the conventional approach used in the AHP-GDM and the results show that the proposed methodology performs more robust manner and calculates the final priorities with smaller MSE than the conventional approach. Other perks of this technique can be listed as follows: it can easily be adapted to any data security standard by updating the elements in the problem, and can be used alone or with any other data security threat analysis methods as a support.

The remainder of this paper is as follows. The relevant theoretical background of the AHP-GDM approach and the Bayesian prioritization procedure for the AHP-GDM is briefly presented in Section 2. In Section 3, an illustrative example is provided to show how the proposed method can be implemented to calculate the relative values of threat incidents. The main results of the illustrative example are also given here. Finally, Section 3 summarizes the conclusions obtained from this study.

2.BACKGROUND

2.1.AHP group decision making (AHP-GDM). The AHP was developed by Saaty [6] in order to deal with problems which involve consideration of multiple criteria simultaneously. It has been extensively applied in complex decision-making problems of choice, prioritization and evaluation. Its ability to synthesize both tangible and intangible characteristics, to accommodate both shared and individual values and monitor the consistency with which a decision-maker makes his judgements made the AHP a widely used multiple criteria decision making (MCDM) tool [19]. The AHP has particular applications in individual and group decision making. According to many researchers AHP is an effective and flexible tool for structuring and solving abstruse group decision situations [15,17,19].

The AHP comprises of four stages: modelling, valuation, prioritization and synthesis. In the modelling stage, a hierarchy which describes the problem is constructed. The overall goal or mission is placed at the top of the hierarchy. The main attributes, criteria and sub-criteria are placed in the subsequent levels below. In the evaluation stage, decision makers compare all the criteria with regard to goal and then all the alternatives with respect to each criterion. Their preferences are included as pairwise comparison matrices in the analysis and they are based on the fundamental scale proposed by Saaty [6]. In the prioritization stage, the local priorities are derived by calculating the eigenvalues of the comparison matrix of each element and global priorities are derived using the hierarchic composition principle. In the last stage, the global priorities for each alternative are synthesized in order to get their total priorities.

There are different methods to accommodate the judgements of decision makers in a group setting [8]. Saaty [16] suggests one of the two methods to proceed: decision makers make each paired comparison individually, or the group is required to achieve accord on each paired comparison. If individual's paired comparison ratio judgements are gathered, the AHP literature describes diverse methods for the prioritization and synthesis processes [6,20,21]. The two conventional procedures to obtain group priorities are the aggregation of individual judgements (AIJ) and the aggregation of individual priorities (AIP). Based on individual judgements, a new judgement matrix is built for the group as a whole in AIJ procedure and the priorities are computed from the new matrix.

In the AIP method, the total priorities are obtained on the basis of individual priorities using one or other aggregation procedure. Synthesis of the model can be done using an aggregation procedure. The weighted geometric mean method is the most commonly used technique for both [22].

2.2. Bayesian prioritization procedure (BPP) for AHP-GDM. Bayesian methods allow the treatment of missing data or incomplete data using data augmentation techniques [23]. The integration of high-dimensional functions has been the major limitation towards the wide application of Bayesian analysis before Markov Chain Monte Carlo (MCMC) methods was introduced.

There are very few references to Bayesian analysis in the AHP literature. [24] provided a Bayesian extension of their regression formulation of the AHP. [25] used MCMC methods to calculate the posterior distributions of judgements and estimated the vector of priorities and the most likely rankings. [15] provided a Bayesian prioritization procedure (BPP) for AHP group decision making that does not require filters for the initial judgements of the decision makers. This procedure is based on the prior assumption of the existence of accord among the decision makers. Unlike the AIJ and the AIP methods, this process uses weightings that are inversely proportional to the decision makers' levels of inconsistency and is more effective when compared to them. This method also can be extended to the case of incomplete pairwise comparison matrices, which is a common problem in complex decision making problems. For such cases, [15] showed that BPP performs much more robust manner than the conventional methods, especially with regard to consistency.

2.2.1. *Statistical model*. Assuming a single criterion, and a set of n alternatives, $A_1,\ldots,A_n$, let $D = D_1,\ldots,D_r$, $r \geq 2$ be a group of r decision makers, each express individual pairwise comparisons with regard to the criterion considered, resulting in r reciprocal judgement matrices, $R^{(k)}$, $k = 1,\ldots,r$ .Their preferences are based on the fundamental scale proposed by Saaty [5]. $R^{(k)} = \left(r_{ij}^{(k)}\right)$ is a positive square matrix $(n \times n)$ which validates $\left(r_{ii}^{(k)}\right) = 1, \left(r_{ij}^{(k)}\right) = 1 \Big/ \left(r_{ji}^{(k)}\right) > 0$ for $i, j = 1,\ldots,n.$ The judgements $\left(r_{ij}^{(k)}\right)$ represent the preference of the decision maker, D_k, when a comparison between A_i and A_j is required. Let $v^G = \left(v_1^G,\ldots,v_n^G\right)$ and $w^G = \left(w_1^G,\ldots,w_n^G\right)$, $w_i^G = v_i^G \Big/ \sum_{j=1}^{n} v_j^G$ be the group's unnormalized and normalized priorities for the alternatives, respectively. As traditionally employed in stochastic AHP [20,24] multiplicative model with lognormal errors is applied in the Bayesian analysis of the model. If the decision makers express all possible judgements, the model will be $r_{ij}^{(k)} = \frac{v_i^G}{v_j^G} e_{ij}^k, i, j = 1,\ldots,n, k = 1,\ldots,r,$

with $e_{ij}^{(k)} \sim LN\left(0, \sigma^{(k)2}\right), i < j$.

Taking the logarithms and eliminating the reciprocal judgements, a regression model with normal errors is obtained given by:

$$y_{ij}^{(k)} = \left(\mu_i^G - \mu_j^G\right) + \in_{ij}^k, i = 1,\ldots,n-1, j = 1,\ldots,n, k = 1,\ldots r$$

Where $\in_{ij}^k \sim N\left(0, \sigma^{(k)2}\right)$ .Here , A_n is established as the benchmark alternative $\left(\mu_n = 0 \Leftrightarrow v_n = 1\right)$. In matrix notation, model can be written as:

$$y^{(k)} = X_{\mu}^G + \in^{(k)}, \text{with } \in^k \sim N\left(0, \sigma^{(k)2} I\right),$$

Where

$$y^{(k)} = \left(y_{12}^{(k)}, y_{13}^{(k)}, \ldots, y_{n-1n}^{(k)}\right)'$$

$X_{t \times n-1} = \left(x_{pq}\right)$ with $x_{pi} = 1, x_{pj} = -1$ and $x_{p\lambda} = 0,$ if $\lambda \neq I, J, \lambda = 1,\ldots,n-1$ and $p = \frac{2n-i}{2}(i-1) + (j-1)$ with $1 \leq i < j \leq n, x_{pi} = 0$ and $x_{p\lambda} = 0$, if $\lambda \neq I, J, \lambda = 1,\ldots,n-1$ and $p = \frac{2n-i}{2}(i-1) + (n-i), \mu^G = \left(\mu_1^G, \mu_2^G, \ldots, \mu_{n-1}^G\right),$ $k = 1,\ldots,r, y^{(k)} = \left(\in_{12}^k, \in_{13}^k, \ldots, \in_{n-1n}^k\right)'$ and $t = n(n-1)\big/2$.

With a constant non-informative distribution as the prior distribution for the vector of log-priorities, μ^G, the posterior distribution of μ^G for complete and precise data is given by:

$$\mu^G \mid y \sim N_{n-1}\left(\hat{\mu}_B, \hat{\Sigma}_B\right),$$

Where $\hat{\mu}_B = \dfrac{\sum_{k=1}^{r} \tau^{(k)} \hat{\mu}^{(k)}}{\sum_{k=1}^{r} \tau^{(k)}}$ and

$$\hat{\Sigma}_B = \left(\Sigma_{k=1}^{r} \tau^{k}\right)^{-1} (X'X)^{-1} \begin{pmatrix} 2/n & 1/n & \dots & 1/n \\ 1/n & 2/n & \dots & 1/n \\ \vdots & \vdots & \ddots & \vdots \\ 1/n & 1/n & \dots & 2/n \end{pmatrix}$$

$\tau(k) = 1 \big/ \sigma^{(k)2}$ and $y = \left(y^{(1)'}, y^{(2)'}, \dots, y^{(r)'}\right)'$.

For the conventional procedure, AIP, the most commonly used method to aggregate group judgements is the geometric mean method. It can be presented as:

$\hat{\mu}_{AIP} = \frac{1}{r} \sum_{k=1}^{r} \hat{\mu}^{(k)}$, where $\hat{\mu}^{(k)} = \left(\hat{\mu}_1^{(k)}, \dots, \hat{\mu}_{n-1}^{(k)}\right)$

with $\hat{\mu}_i^{(k)} = \bar{y}_i^{(k)} - \bar{y}_n^{(k)}$.

The other conventional procedure, AIJ, is not mentioned in this study since [15] showed that it gives almost the same results with the AIP method. Further data and theorems can also be found in [15].

2.2.2. Incomplete data. Most MCDM methods are based on the assumption that complete data about the model parameters (scores, attribute weights) need to be elicited as 'exact' point estimates [26]. According to [27], decision makers might provide only incomplete data in real life. The reasons for the incomplete data are as follows: (1) a decision might be made under pressure of limited time and lack of data; (2) many of the attributes might be intangible or non-monetary because they reflect social and environmental impacts; (3) decision makers might have limited attention and data processing capabilities; and (4) all participants might not have equal expertise about the problem domain in group settings. As a consequence, all of the decision makers may not express the $n \times (n-1) \big/ 2$ possible judgements in the reciprocal pairwise comparison matrix or may express inconsistent judgements. There are many methods proposed to overcome this problem (see [26] for more data). BPP can also naturally be extended to the case of incomplete data, where it performs more robust manner compared with the conventional methods in terms of consistency. In such cases, the equations of model (3) could be expressed as:

$$y^{(k)} = X_{\mu}^{G} + \varepsilon^{(k)},$$

With $\varepsilon^{(k)} \sim N_{t_k}\left(0, \sigma_k^2 I_{t_k}\right), k = 1, \dots, r;$ and in the matrix form it can be expressed as:

$$y = X\left(1_r \otimes I_{n-1}\right)\mu^G + \varepsilon \quad \text{with} \quad \varepsilon \sim N_t(0, D)$$

where

$$y = \left(y^{(1)'}, y^{(2)'}, \dots, y^{(r)'}\right)',$$

$$X = diag\left(X^{(1)}, X^{(2)}, \dots, X^{(r)}\right), \varepsilon = \left(\varepsilon^{(1)}, \varepsilon^{(2)}, \dots, \varepsilon^{(r)}\right)'$$

and

$$D = diag\left(\sigma^{(1)2} I_{t_1}, \dots, \sigma^{(r)2} I_{t_r}\right). 1_r = (1, 1, \dots, 1)', t_k$$

is the number of judgements issued by each decision maker $D_k, t = t_1 + \dots + t_r$

by all decision makers and $\otimes$ denotes the Kronecker product.

With a constant non-informative distribution as the prior distribution for the vector of log-priorities. $\left(\mu^G\right)$, the posterior distribution of μ^G for incomplete and precise data is given by:

$$\mu \mid y \sim N_{n-1}\left(\hat{\mu}_B, \hat{\Sigma}_B\right),$$

Where

$$\hat{\mu}_B = \left(\sum_{k=1}^{r} \tau^{(k)} X^{(k)'} X^{(k)}\right)^{-1} \left(\sum_{k=1}^{r} r^{(k)} X^{(k)'} y^{(k)}\right)^{-1}$$

$$= \left(\left(1_r \otimes I_{n-1}\right)\left(X'D^{-1}X\right)\left(1_r \otimes I_{n-1}\right)\right)^{-1} \left(1_r \otimes I_{n-1}\right)\left(X'D^{-1}y\right),$$

$$\hat{\Sigma}_B = \left(\sum_{k=1}^{r} \tau^{(k)} X^{k'} X^{k} \right)^{-1}.$$

The estimator of μ^G obtained by means of the AIP procedure is given by:

$$\hat{\mu}_{AIP} = \frac{1}{r} \sum_{k=1}^{r} \hat{\mu}^{(k)} = \frac{1}{r} \sum_{k=1}^{r} \left(X^{(k)'} X^{(k)} \right)^{-1}, \left(X^{(k)'} y^{(k)} \right)$$

$$= \frac{1}{r} \left(1_r \otimes I_{n-1} \right) (X'X)^{-1} (X'y).$$

3.CONCLUSION

Threat management requires the use of more flexible approaches to measure data security threat. The AHP-GDM offers a technical support for threat analysis by obtaining the judgements of managers and systematically calculating the relative threat values.

The AHP-GDM is a powerful technique that is easy to understand and simple to operate. It is a flexible and practical tool for any organization to rank the threat incidents recurrently. However, there might be some complexities to use the AHP-GDM in threat evaluation. Decision makers participated in the analysis may have limited expertise about the problem domain or the AHP analysis. Also, they may have difficulties to make pairwise comparisons efficiently because of the large number of assets, threats and vulnerabilities which could result in incomplete or inconsistent judgements.

Considering the problems mentioned above, we propose using BPP based AHP for data security threat evaluation. It is assumed that agreement exists among the decision makers with regard to the priorities for each element in this decision system. The multiplicative model with log-normal errors is applied to the problem and the Bayesian analysis is used. This is a process of weighted aggregation of individual priorities and the weights are inversely proportional to the decision makers' levels of inconsistency. We compared the method with the orthodox approaches used in the AHP-GDM.

The results show that the proposed methodology performs more robust manner and calculates the final priorities with smaller MSE than the traditional approach. So, it can be concluded that the proposed methodology aggregate the individuals' judgements more effectively than the conventional method, especially after omitting the inconsistent judgements in the pairwise comparison matrices. This method provides managers a flexible way to express their judgements, without forcing them to give complete, consistent and congruent judgements and letting them completely focus on the threat management itself. Moreover, it serves the practitioner since the judgements of decision makers directly enter the analysis without any reducing or filtering process.

Any organization can easily adapt this method to their data security system by updating all the elements in the illustrative model, i.e., list of most valuable data assets, threats and vulnerabilities. This technique could be used alone or with any other data security threat analysis methods as a support; and can easily be adapted to any data security standard.

In this study, we applied BPP based AHP to prioritize and order threat incidents which could satisfy the aim of threat management. This approach can also be used for many multiple criteria group decision making problems such as project selection, facility location selection, supplier selection or evaluation, diagnosis and treatment selection for disease management, financial decision making and crisis forecasting, and evacuation selection for emergency management.

Our study is based on the model from a non-informative Bayesian standpoint, where the variances of error terms represented by the inconsistency levels of decision makers are assumed to be known. In the future, this approach can be extended by taking the variances of error terms as additional parameters, or by implementing an informative Bayesian model in which a good estimate of prior distribution for the vector of log priorities is used.

This study is based on two assumptions. The first assumption is that there is a accord among the decision makers. Gargallo et al. [28] proposed a Bayesian estimation procedure to determine the priorities where a prior accord among them is not required.
The second assumption is that there is no interaction or dependence between the elements in the decision system. We are currently working on the situations where this assumption is unsatisfied.

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