Performance Analysis of Fast wavelet transform and Discrete wavelet transform in Medical Images using Haar, Symlets and Biorthogonal wavelets

Sandeep Kaur[1],Gaganpreet Kaur[2],Dheerendra Singh[3]

[1]*Student Masters Of Technology, Shri Guru Granth Sahib World University, Fatehgarh Sahib*

[2]*Assistant Professor, Shri Guru Granth Sahib World University, Fatehgarh Sahib*

[3]*Professor & Head, SUSCET, Tangori*

Abstract- Data compression is the technique to reduce the redundancies and irrelevancies in data representation in order to decrease data storage requirements and hence communication costs. Reducing the storage requirement is equivalent to increasing the capacity of the storage medium and hence communication bandwidth. The objective of this paper is to compare a set of different wavelets for image compression. Image compression using wavelet transforms results in an improved compression ratio, PSNR and Elapsed time is compared using various wavelet families such as Haar, Symlets and Biorthogonal using Discrete Wavelet Transform and Fast wavelet transform. In DWT wavelets are discretely sampled. The Discrete Wavelet Transform analyzes the signal at different frequency bands with different resolutions by decomposing the signal into an approximation and detail information. The Fast wavelet transform has the advantages over DWT is higher compression ratio and fast processing time using different wavelets.The study compares DWT and FWT approach in terms of PSNR, Compression Ratios and elapsed time for different Images. Complete analysis is performed at second and third level of decomposition using Haar Wavelet, Symlets wavelet and Biorthogonal wavelet using medical images.

Keywords: Discrete Wavelet Transform, Fast Wavelet Transform, Approximation and Detail Coefficients, Haar, Symlets

I. Introduction

The discrete wavelet transform (DWT) refers to wavelet transforms for which the wavelets are discretely sampled [1]. A transform which localizes a function both in space and scaling and has some desirable properties compared to the Fourier transform. The transform is based on a wavelet matrix, which can be computed more quickly than Fourier matrix. DWT has various advantages over DCT it removes the problem of blocking artifact that occur in DCT. DWT provides better result at higher compression ratio as compare to DCT.

Various characteristics of DWT as follow:

a) It allows image multi resolution representation in a natural way because in this more wavelet subbands are used to progressively enlarge the low frequency subbands

b) It supports wavelet coefficients analysis in both space and frequency domains.

c) For natural images, the DWT achieves high compactness of energy in the lower frequency subbands, which is extremely useful in applications such as image compression.

A. Lossy and Lossless Compression:

The objective of image compression is to reduce redundancy of the image data in order to be able to store or transmit data in an efficient form. Image compression can be lossy or lossless. [1]Lossless compression in which no data is loss means in this less compression and more information preserved. Lossless compression is sometimes preferred for artificial images such as technical drawings, icons or comics. This is because lossy compression methods, especially when used at low bit rates, introduce compression artifacts. [2] Lossy compression is that in which compression is high but less information is preserved as compare to lossless compression. Lossy methods are especially suitable for natural images such as photos in applications where minor loss of fidelity is acceptable to achieve a substantial reduction in bit rate.

The JPEG 2000 standard proposes a wavelet transform stage since it offers better rate/distortion (R/D) performance than the traditional discrete cosine transform (DCT). Unfortunately, despite the benefits that the wavelet transform entails, some other problems are introduced. Wavelet-based image processing systems are typically implemented by memory-intensive algorithms with higher execution time than other transforms. In the usual DWT implementation [2], the image decomposition is computed by means of a convolution filtering process and as the filter length increases its complexity rises.

II. Discrete Wavelet Transform

Wavelet transform (WT) represents an image as a sum of wavelet functions (wavelets) with different locations and scales [3]. Any decomposition of an image into wavelets involves a pair of waveforms: one to represent the high frequencies corresponding to the detailed parts of an image (wavelet function ψ) and one for the low frequencies or smooth parts of an image (scaling function Ø). DWT is a multi resolution decomposition scheme for input signals. The original signals are decomposed into two subspaces, low-frequency (low-pass) subband and high-frequency (high-pass) subband. For the classical DWT, the forward decomposition of a signal is implemented by a low-pass digital filter H and a high-pass digital filter G. Both digital filters are derived using the scaling function Φ(t) and the corresponding wavelets Ψ(t). The system down samples the signal to half of the filtered results in the decomposition process. If the four-tap and non-recursive FIR filters with length L are considered, the transfer functions of H and G can be represented as follows

$H(z) = h_0 + h1z_{-1} + h2z_{-2} + h3z_{-3}$ (3)

$G(z) = g_0 + g1z_{-1} + g2z_{-2} + g3z_{-3}$ (4)

The discrete wavelet transform has a huge number of applications in Science, Engineering, Mathematics and Computer Science. Wavelet compression is a form of data compression well suited for image compression (sometimes also video compression and audio compression). First a wavelet transform is applied. This produces as many coefficients as there are pixels in the image (i.e.: there is no compression yet since it is only a transform). These coefficients can then be compressed more easily because the information is statistically concentrated in just a few coefficients. This principle is called transform coding. After that, the coefficients are quantized and the quantized values are entropy encoded and/or run length encoded[4].

III. Fast Wavelet Transform

In 1988, Mallat produced a fast wavelet decomposition and reconstruction algorithm. The Mallat algorithm for discrete wavelet transform (DWT) is, in fact, a classical scheme in the signal processing community, known as a two-channel subband coder using conjugate quadrature filters or quadrature mirror filters (QMFs).

(a) The decomposition algorithm starts with signal *s*, next calculates the coordinates of A_1 and D_1, and then those of A_2 and D_2, and so on.

(b) The reconstruction algorithm called the inverse discrete wavelet transform (IDWT) starts from the coordinates of A_J and D_J, next calculates the coordinates of A_{J-1}, and then using the coordinates of A_{J-1} and D_{J-1} calculates those of A_{J-2}, and so on.

In order to understand the multiresolution analysis concept based on Mallat's algorithm it is very useful to represent the wavelet transform as a pyramid, as shown in figure 1. The basis of the pyramid is the original image, with C columns and R rows.

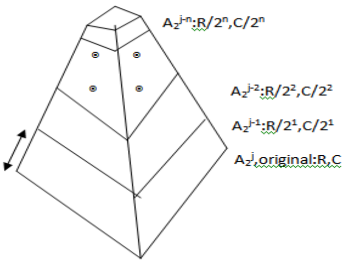
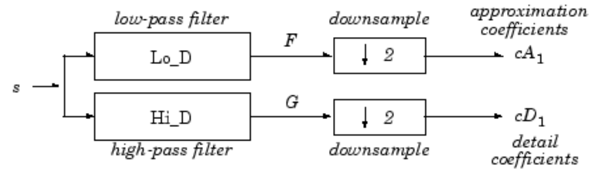


Figure 1: Pyramidal representation of Mallat's wavelet decomposition algorithm.

Given a signal *s* of length *N*, the DWT consists of $\log_2 N$ stages at most. Starting from *s*, the first step produces two sets of coefficients: approximation coefficients cA_1, and detail coefficients cD_1. These vectors are obtained by convolving *s* with the low-pass filter Lo_D for approximation, and with the high-pass filter Hi_D for detail, followed by dyadic decimation.



The length of each filter is equal to $2n$. If N = length (s), the signals F and G are of length $N + 2n - 1$, and then the coefficients cA_1 and cD_1 are of length

$$\text{floor}\left(\frac{(N-1)}{2}+n\right)$$

The next step splits the approximation coefficients cA_1 in two parts using the same scheme, replacing *s* by cA_1 and producing cA_2 and cD_2, and so on.

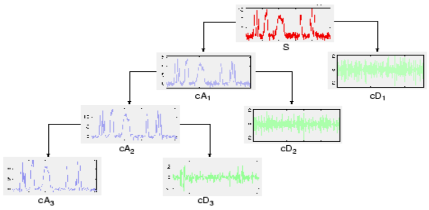
Table1: Comparison between DWT and FWT

DWT	FWT
(a) DWT has the problems of border distortions (b) It takes large time for processing (c) DWT has low compression ratio using different wavelets.	(a) FWT removes the problem of border distortion (b) It takes a less time for processing. (c) FWT has higher compression ratio using different wavelets

IV. Multilevel Decomposition

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower resolution components. This is called the wavelet decomposition tree. Wavelet decomposition tree which starts with root 's' represents a signal that decomposed into approximation and detail coefficients in which approximation represent the smooth part of the image and detail part of the image represent the redundancy or noisy part of the image that is discarded by each iteration of decomposition process.

Figure 2: Multilev



el

Decomposition

Lifting schema of DWT has been recognized as a faster approach
(a) The basic principle is to factorize the polyphase matrix of a wavelet filter into a sequence of alternating upper and lower triangular matrices and a diagonal matrix.
(b) This leads to the wavelet implementation by means of banded-matrix multiplications
Algorithm follows a quantization approach that divides the input image in 4 filter coefficients as shown below, and then performs further quantization on the lower order filter or window of the previous step. This quantization depends upon the decomposition levels and maximum numbers of decomposition levels to be entered are 3 for DWT [5].

A. *Wavelet Reconstruction*

The filtering part of the reconstruction process is important because it is the choice of filters that is crucial in achieving perfect reconstruction of the original signal. The down sampling of the signal components performed during the decomposition phase introduces a distortion called aliasing. It turns out that by carefully choosing filters for the decomposition and reconstruction phases that are closely related (but not identical), we can "cancel out" the effects of aliasing [6].

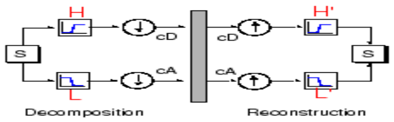


Figure 3: Wavelet Reconstruction

V. Wavelet Families

A wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. This is also known as one complete cycle it not only has an oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet[7].Several families of wavelets that have proven to be especially useful are included in the wavelet toolbox. This paper has used Haar, Symlets and Biorthogonal wavelets for image compression. The details of the Haar, Symlets and Biorthogonal Wavelet are shown below:

A. *Haar Wavelets*

Haar wavelet is the first and simplest. Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1.

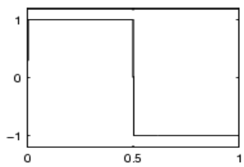


Figure 4: Haar Wavelet Function Waveform

B. *Symlet Wavelets*

The Symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. There are 7 different Symlets functions from sym2 to sym8. Wavelet functions of Symlets wavelet is psi.

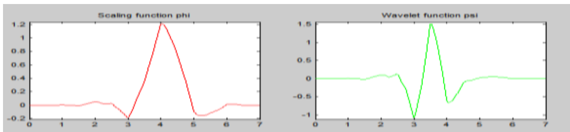


Figure 5: Sym4 wavelet

C. *Biorthogonal Wavelets*

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties are derived. General characteristics of Biorthogonal wavelet as follows:

(a) Compactly supported
(b) Biorthogonal spline wavelets for which
(c) Symmetry and exact reconstruction are possible
(d) Family Biorthogonal
(e) Short name bior
(f) Order Nr,Nd Nr = 1 , Nd = 1, 3, 5
(g) r for reconstruction Nr = 2 , Nd = 2, 4, 6, 8
(h) d for decomposition Nr = 3 , Nd = 1, 3, 5, 7, 9

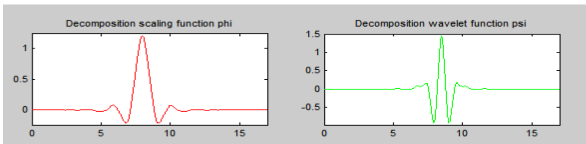


Figure 6: Bior6.8 wavelet

VI. Performance Parameters

In this work input image compressed to a certain level using DWT / FWT based lifting and quantization scheme explained above by maintaining a good signal to noise ratio. Quantitative analysis have been presented by measuring the values of attained Peak Signal to Noise Ratio and Compression Ratio at different decomposition levels. The intermediate image decomposition windows from various low pass and high pass filters.

(a)PSNR:

PSNR is most commonly used to measure the quality of reconstruction of lossy compression codecs (e.g., for image compression). The signal in this case is the original data, and the noise is the error introduced by compression. When comparing compression codecs, PSNR is an approximation to human perception of reconstruction quality. Although a higher PSNR generally indicates that the reconstruction is of higher quality, in some cases it may not.

(a) PSNR is defined as:

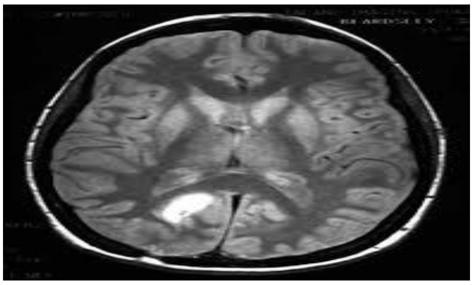
$$PSNR = 10 \cdot \log_{10}\left(\frac{MAX_I^2}{MSE}\right) = 20 \cdot \log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right) = 20 \cdot \log_{10}(MAX_I) - 10 \cdot \log_{10}(MSE)$$

(b) Compression Ratio:

Ratio of the size of compressed image to the input image is often called as compression ratio.

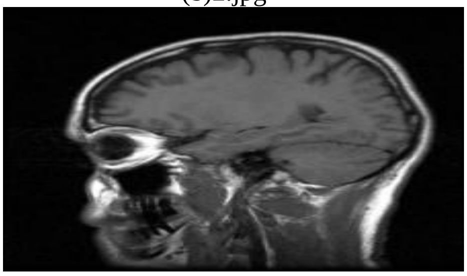
VII. Design and Implementation

FWT and DWT technique is used for obtain the desired results. Different wavelets are used at 2nd and 3rd level of decomposition and comparative analysis of Haar, Symlets and Biorthogonal family is displayed. Quantitative analysis has been presented by measuring the values of attained Peak Signal to Noise Ratio and Compression Ratio at 2nd and 3rd decomposition levels. The intermediate image decomposition windows from various low pass and high pass filters. Qualitative analysis has been performed by obtaining the compressed version of the input image by FWT and DWT Techniques and comparing it with the test images. Our results shows that Haar wavelet gives better result for all images in FWT as compare to DWT in terms of compression ratio, PSNR value and takes less time for compression. Also we found that for all three images FWT takes less time using Haar, sym4 and bior6.8 as compare to DWT using Haar, sym4 and bior6.8.



(a)1.jpg

(b)2.jpg

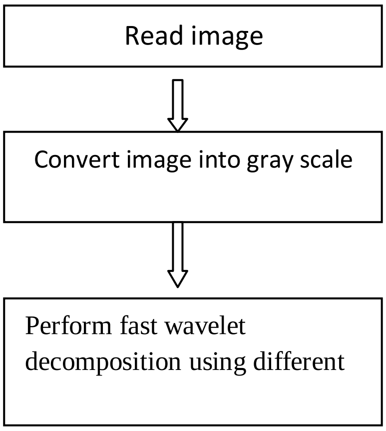


(c)3.jpg

Figure 7: Data Set of MRI Images used in the work

A. *Diagrammatic design of proposed work*

Diagram shows the basic steps followed to obtain the results using FWT and DWT for comparison.



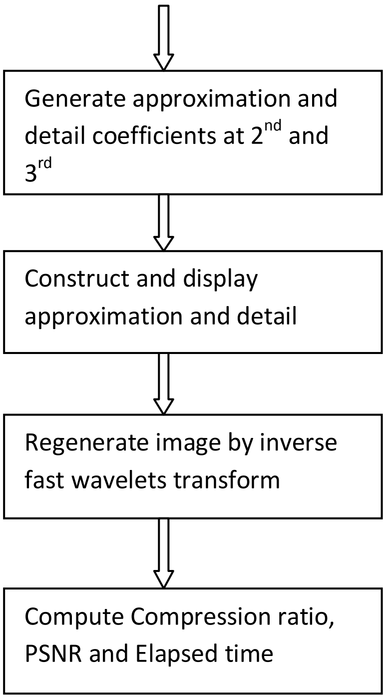


Figure 8: Basic block diagram

Results:The results below shows the original image , compressed image and reconstructed image with2nd level and 3rd level decomposition using FWT with Haar wavelet,sym4 wavelet and bior6.8 wavelets. The reconstructed image is approximate of original image when viewed with eye. Table 1 and Table.2 shows the results obtained by using FWT and

DWT in terms of PSNR and compression Ratios at 2nd and 3rd levels of decompositions. Also, the elapsed time for Haar , sym4 and bior6.8 at 2nd and 3rd level of decomposition have been obtained using different images and show below in tabular form:

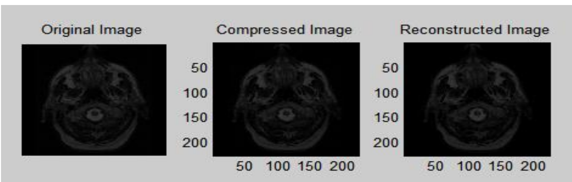


Figure 9: 1.jpg image at 2nd level of decomposition with Haar wavelet

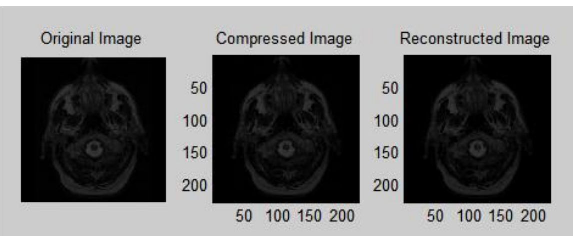


Figure 10: 1.jpg image at 2nd level of decomposition with sym4 wavelet

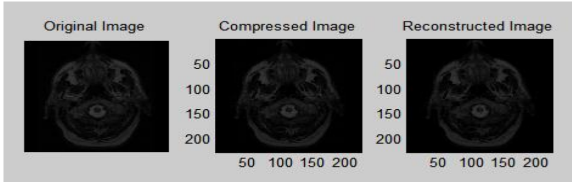


Figure 11: 1.jpg image at 2nd level of decomposition with bior6.8wavelet

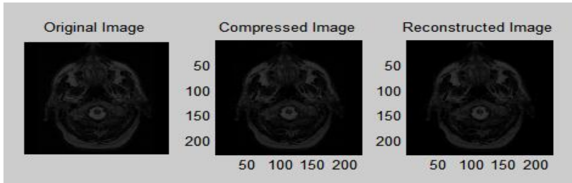


Figure 12: 1.jpg image at 3rd level of decomposition with Haar wavelet

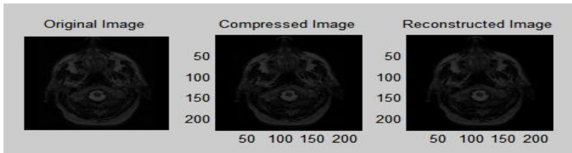


Figure 13: 1.jpg image at 3rd level of decomposition with sym4 wavelet

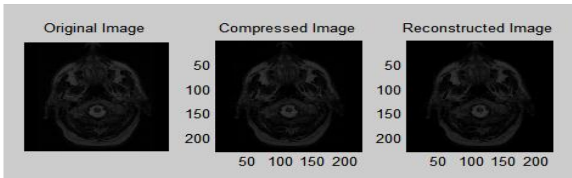


Figure 14: 1.jpg image at 3rd level of decomposition with bior6.8 wavelet

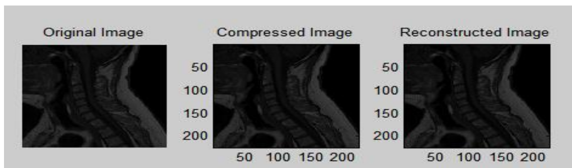


Figure 15: 2.jpg image at 2nd level of decomposition with Haar wavelet

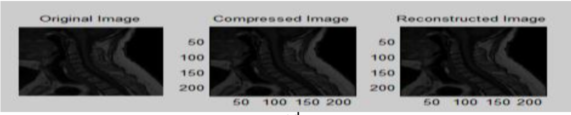


Figure 16: 2.jpg image at 2nd level of decomposition with sym4 wavelet

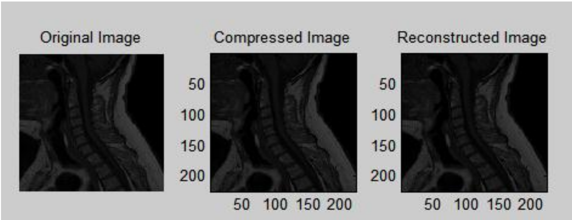


Figure 17: 2.jpg image at 2nd level of decomposition with bior6.8wavelet

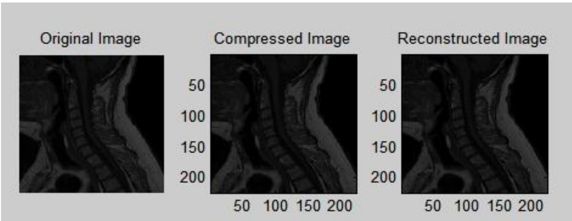


Figure 18: 2.jpg image at 3rd level of decomposition with bior6.8 wavelet

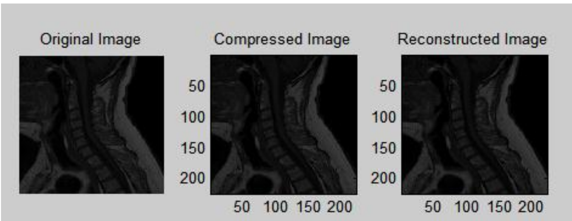


Figure 19: 2.jpg image at 3rd level of decomposition with bior6.8 wavelet

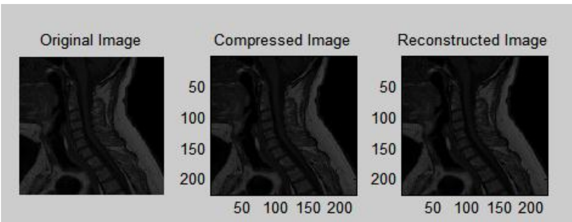


Figure 20: 2.jpg image at 3rd level of decomposition with bior6.8 wavelet

Following figures shows the comparative analysis of Compression ratio(CR) and elapsed time usin DWT and FWT.

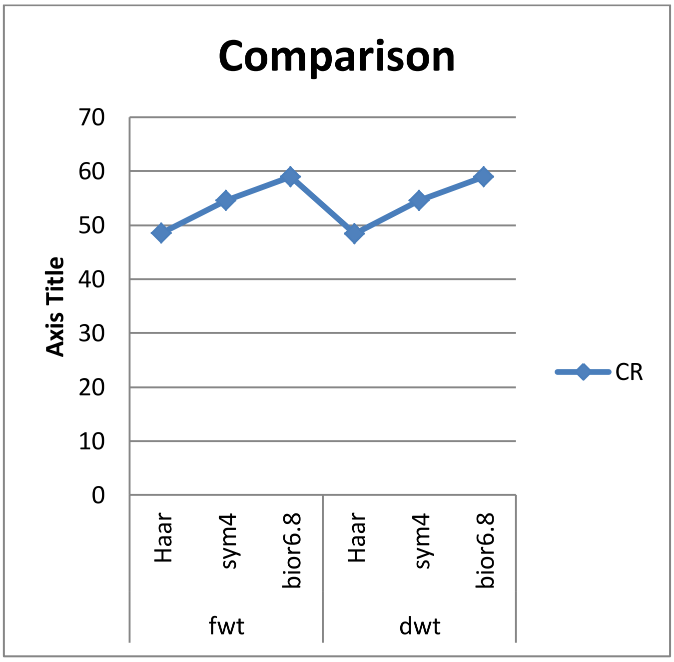


Figure 21: Graph shows comparison of compression ratio(CR) of 1.jpg image at 3rd level

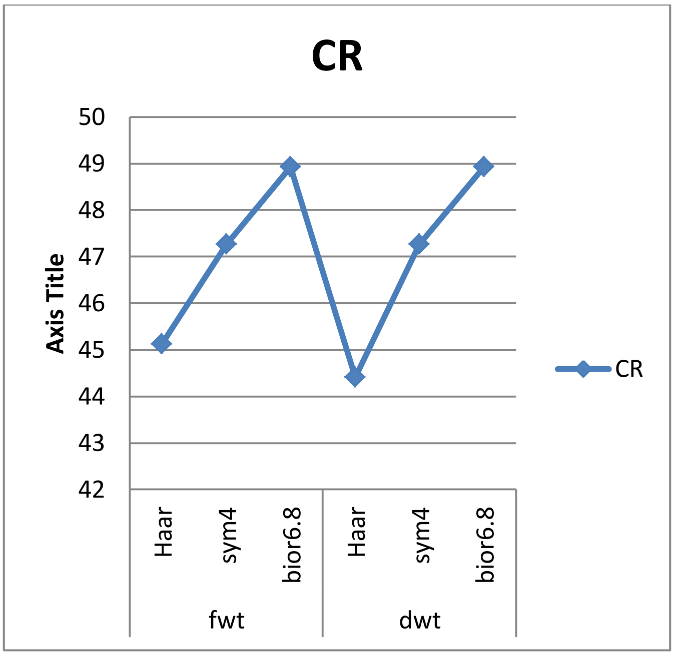


Figure 22: Graph shows comparison of compression ratio (CR) of 2.jpg image at 3rd level

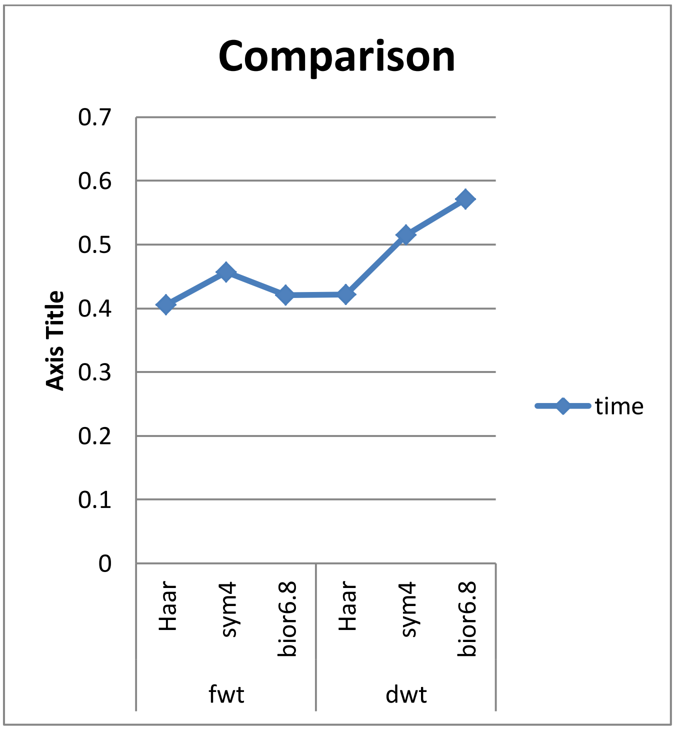


Figure 23: Graph shows comparison of Elapsed time of 1.jpg image at 3rd level

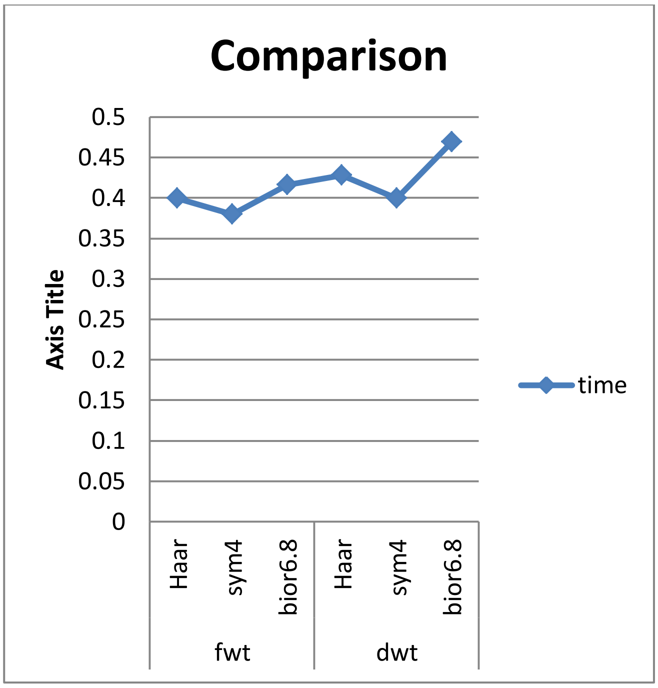


Table2: Image 2.jpg in figure 7.1(b)

Figure 24: Graph shows the comparative analysis of time of 2.jpg image at 3rd level

Table 1: Image 1.jpg in figure 7.1(a)

Wave lets	Decomp osition level (input)	Tech nique	Compress ion ratio output in %	PSNR output in %	Elapsed time (sec)
Haar	2	**FWT**	**47.5255**	**99.9984**	**0.366792**
		DWT	47.4553	99.9985	0.471094
	3	**FWT**	**48.4927**	**99.9983**	**0.406150**
		DWT	48.4208	99.9984	0.421460
Sym4	2	**FWT**	**53.4243**	**99.9941**	**0.396377**
		DWT	53.4243	99.9941	0.445243
	3	**FWT**	**54.5259**	**99.9938**	**0.456489**
		DWT	54.5259	99.9938	0.514927
Bior6 .8	2	**FWT**	**56.8393**	**99.9936**	**0.384457**
		DWT	56.8393	99.9936	0.431324
	3	**FWT**	**58.9769**	**99.9934**	**0.420637**
		DWT	58.9769	99.9934	0.571778

Wavelets	Decomposition level(input)	Technique	Compression ratio output in %	PSNR Output in %	Elapsed time(sec)
Haar	2	**FWT**	**44.7664**	**99.9815**	**0.307433**
		DWT	44.0629	99.9825	0.367260
	3	**FWT**	**45.1192**	**99.9813**	**0.399727**
		DWT	44.4137	99.9824	0.427538
Sym4	2	**FWT**	**46.9657**	**99.9875**	**0.312167**
		DWT	46.9657	99.9875	0.405976
	3	**FWT**	**47.2684**	**99.9896**	**0.380048**
		DWT	47.2684	99.9896	0.399823
Bior6.8	2	**FWT**	**49.2544**	**99.9880**	**0.329967**
		DWT	49.2544	99.9880	0.378912
	3	**FWT**	**48.9225**	**99.9920**	**0.416296**
		DWT	48.9225	99.9920	0.468932

VIII. Conclusion and future scope

Image Compression is performed in the MATLAB software using wavelet toolbox. DWT and FWT based compression techniques have been implemented using lifting scheme and their results have been displayed in terms of qualitative analysis using image visual quality of input image, compressed image and reconstructed image and Quantitative analysis have been performed in terms of PSNR, compression ratio and Elapsed time for both DWT and FWT at second level and third level of decomposition using Haar Wavelets, Symlets and Biorthogonal wavelets. The result have shown that FWT gives better compression ratio as compare to DWT and takes less time for compression using Haar wavelet. Picture visual quality or PSNR achieved with fast wavelet transform is slightly similar than that of discrete wavelet transform technique but the compression ratio achieved with fast wavelet transform is more than that of discrete wavelet transform technique.

References

[1] Zigong Gao, F.Zheng Yuan, *"Quality Constrained Compression Using DWT Based Image Quality Metric"*,IEEE Trans,September 10,2007.

[2] Sonja Grgic, Kresimir Kers, Mislav Grgc, "*Image Compression using Wavelets*", University of Zagreb, IEEE publication, 1999.

[3]Sonja Grgic, Mislav Grgic,*"Performance Analysis of Image Compression Using Wavelets"*,IEEE Trans,Vol.48,No.3,June 2001.

[4] Jashanbir Singh, Reecha Sharma,*"Comparativ performance analysis of Haar,Symlets and Bior wavelets on image compression using Discrete wavelet Transform"*, International journal of Computers and Dstributed Systems, Volume 1,Issue 2,August,2012.

[5] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "*Image coding using wavelet transform"*, IEEE Trans. Image Processing, vol. 1, pp.205-220, 1992.

[6] P.L. Dragotti, G. Poggi, and A.R.P. Ragozini, "*Compression of multispectral images by three-dimensional SPIHT algorithm"*, IEEE Trans. on Geoscience and remote sensing, vol. 38, No. 1, Jan 2000.

[7] Sarita Kumari, Ritu Vijay, "*Analysis of Orthogonal and Biorthogonal Wavelet Filters for Image Compression*", International Journal of Computer Applications , Volume 21– No.5, May 2011.

[8] B. Kim and W.A. Pearlman, "*An embedded wavelet video coder using three-dimensional set partitioning in hierarchical tree"*, IEEE Data Compression Conference, pp.251-260, March 1997.

[9]V Kumar, V Sunil, "*Image Compression Techniques by using Wavelet Transform"*, Journal of information engineering and applications, Vol 2, No.5, 2012.

[10]Anil kumar Katharotiya, Swati Patel,*"Comparative Analysis between DCT & DWT Techniques of Image Compression"*, Journal of information engineering and applications, Vol 1, No.2, 2011.

[11] S. Mallat, "*Multifrequency channel decompositions of images and wavelet models"*, IEEE Trans. Speech, Signal Processing, vol. 37, pp.2091-2110, Dec. 1989.

[12] A.N. Netravali and B.G. Haskell, *Digital pictures, representation and compression*, in Image Processing, Proc. of Data Compression Conference, pp.252-260, 1997.

[13] E. Ordentlich, M. Weinberger, and G. Seroussi, "*A low-complexity modeling approach for embedded coding of wavelet coef_cients"*, in Proc. IEEE Data Compression Conf., Snowbird, UT, pp. 408-417, Mar. 1998.

[14] A. Said and W.A. Pearlman, "*A new, fast and ef_cient image codec based on set partitioning in hierarchical trees"*, IEEE Trans. on Circuits and Systems for Video Technology 6, pp. 243-250, June 1996.

[15] J.M. Shapiro, "*Embedded image coding using zerotrees of wavelet coef_cients"*, IEEE Trans. Signal Processing, vol. 41, pp.3445-3462, Dec. 1993.

[16] D. Taubman, "*High performance scalable image compression with EBCOT"*, IEEE Trans. on Image Processing, vol. 9, pp.1158-1170, July,2000.

[17] I.H. Witten, R.M. Neal, and J.G. Cleary, "*Arithmetic coding for data compression"*, Commun. ACM, vol. 30, pp. 520-540, June 1987.

[18] J.W. Woods and T. Naveen, "*Alter based bit allocation scheme for subband compression of HDTV"*, IEEE Transactions on Image Processing, IP-1:436-440, July 1992.