Conditional Activity-Based Model for Describing User Experience Using Category Theory

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***Abstract*— Designing a user experience is a critical issue for building professional and efficient systems. The user experience introduces new research activities that focused on the interactions between products, applications, designers, and users. To achieve specific user experience goals, we need to find an optimal model for representing this user experience which includes behavior and emotions experiences in efficient and helpful way. In this paper, we propose a mathematical activity-based model for describing user experience including his behavior and emotions experiences by using category theory elements. This model describes any user behavior as an activity that consists of a set of actions and operations where their existence is based on a set of conditions. Also, we introduce a conditional category model that uses this activity representation by modifying category theory based on conditional criteria. This new model is presented as an efficient tool for representing user experience and his/her behavior and will be helpful scheme in building professional products and applications.**

***Keywords*— Human computer interaction, User experience design, Category theory, Activity theory, Mathematical modelling.**

I. Introduction

With the development of information technology schemes, HCI techniques and electronic devices, designing a user experience (UX) became a critical issue for building professional and efficient systems. The user experience introduces new research activities that focused on the interactions between products, applications, designers, and users. Recently, a lot of industrial and technological companies have touched the importance of UX as a key success issue in product design [1]. The creating meaningful UX is not just usability but it goes far more. Therefore, it is essential to take into account other cognitive, socio-cognitive, and affective aspects of UX in the interaction process, such as users' enjoyment, brand loyalty, mental models, and aesthetic experience [2]. In addition, the user behavior is very important issue to be considered in designing UX.

In product design process, there are many interdependent designing attributes are considered as a consistent whole to create unique UX, especially to achieve valuable higher economic benefit and customer desires [3]. The evolution of user's emotional states and cognitive processes with choice decision making are the chain of human-product interactions [4]. Traditionally, most of designers concentrated on functional requirements for physical products and did not consider users' behavior and affective and cognitive needs. Recently, designers can utilize the new technologies, compose multimedia platforms with services, or use of sensory information for creating meaningful UX based on the context of work environments [5].

In decision making, human emotional experience play a significant role towards product success [6]. Therefore, it is very important to consider the human dimension in design research [7]. Most of existing human decision making systems have been well addressed based on the user cognitive experiences. However, these systems miss the affective elements for modeling, analyzing and simulating human realization on UX in the predominant computational models [8]. Recent models based on behavioral decision theories focus on cognitive errors and heuristics in human decision making, but still ignore the role of emotion in human decision making [0]. Users' affective states often influence their experience at the time of decision making, So a single cognitive perspective is not optimal for analyzing human decisions for meaningful UX [10]. Recently, in [11] the integration of emotion and cognition has been driven by the intimate coupling of affective and cognitive decisions.

To achieve specific user experience goals, we need to find an optimal model for representing this user experience and his/her behavior in efficient and helpful way. In this paper, we propose a mathematical activity-based model for describing user experience and his behavior by using category theory elements. This model describes any user behavior as an activity that consists of a set of actions and operations where their existence is based on a set of conditions. Also, we introduce a conditional category model that uses this activity representation by modifying category theory based on conditional criteria. This new model is presented as an efficient tool for representing user experience and his/her behavior and will be helpful scheme in building professional products and applications.

The rest of the paper is organized as follows. Section 2 includes a detailed survey of the related work. Section 3 describes the user behavior/experience design problem. Section 4 exhibits the details of the proposed mathematical activity-based model. Case study and related discussion are presented in Section 5. Finally, Section 6 concludes the paper.

II. Related Work

Most of user-centered design researchers were intensified affective perception of a product use and concentrated on a product functionality and usability aspects. However, they gave a slight concern of how affect influences behavior and emotion experiences of a user as a whole.

A lot of models were formulated for analyzing and predicting choice behavior and preference of a user in a variety of application contexts [12]. The quality function deployment, *QFD*, is one of the most commonly used methods for representing user preferences in engineering design [13]. To map product features and functionality favored by users, a house of quality is formulated. Also, to deal with the uncertainty aspects, fuzzy set are integrated [14]. For example, in [15] to design the B787 Dreamliner commercial aircraft the QFD was used to translate lifestyle, image, and psychological needs into design requirements. To understand the basic human needs for human experience design, there is increasing trend for studying of interaction between affect and cognition. For instance, Lisetti and Nasoz [16] examined that how affect interacts with cognition and developed a multimodal affective user interface for simulating human intelligence. An affective-cognitive decision framework was proposed for learning and decision making in [10]. In [17], the authors deducted that affect and cognition are highly interdependent because the phenomena themselves are coupled

In [18], for designing a healthcare system a think-aloud protocol was applied to investigate cognitive requirements of nurses and physicians. To improve affective UX, there are a lot of research areas gave more attention for using a user affect an emotions such as users' imaginary expectation and momentary emotions in different contexts and at different points of time [19], [20]. In [21], the affective UX (AUX) was improved by selecting appropriate design elements that are able to extract positive emotions.

Most of existing works can build meaningful UX model based on a user behavior or a user emotion separately and may not build a meaningful UX model based on both of them together. So, In this paper, we will propose an architecture model which combine a user behavior and a user emotion in one model to design more efficient UX model. Also, none of them can not represent a user behavior or a user emotion by using a unified model in UX design. So, we will propose a mathematical categorical representation form to represent both of them in an unified representation model by using category theory [22].

III. User Experience Design Problem

In this section, we will introduce our proposed definitions, assumptions, and models then we will describe the UXD problem.

A. Definitions, Assumptions, and Models

We define a user experience design process as a quadruple system $Q(AT,AC,OP,CN)$ where, $AT = \{av_i : 1 \leq i \leq M\}$ is a set of all activities in the system and M is the total number of activities in the system, $AC = \{ac_j : 1 \leq j \leq N\}$ is a set of all actions in the system and N is the total number of activities in the system , $OP = \{op_k : 1 \leq k \leq K\}$ is a set of all operations in the system and K is the total number of operations in the system, and $CN = \{cn_l : 1 \leq l \leq L\}$ and L is the total number of logical conditions in the system. Also, we assume that each operation $op_k \in OP$ consists of a set of sequential tasks which is denoted as $TS(op_k) = \{ts_r : 1 \leq r \leq R\}$ where R is the total number of tasks in operation op_k. We denote a set of user emotions as $EN = \{en_s : 1 \leq s \leq S\}$ where S is the total number of detected user emotions.

B. UXD Problem Description

The user behavior and emotion experiences are the main issues to describe user experience. So, the user experience design (UXD) problem is how to describe both of two issues in a unified model efficiently. The main objective of this model is achieving all of systems goals in a professional and helpful way. Such that this model must satisfy all related conditions of the system. Therefore, based on our assumptions and system models, we can describe the UXD problem as follows.

Objective: Find a Unified Description Model UDM(Q).
Such that:

(1) $\forall\, av_i \in AT, \exists UE(av_i)\, describe\ av_i\ in\ UDM(Q)$

(2) $\forall\, ac_j \in AC, \exists UE(ac_j)\, describe\ ac_j\ in\ UDM(Q)$

(3) $\forall\, op_k \in OP, \exists UE(op_k)\, describe\ op_k\ in\ UDM(Q)$

(4) $\forall\, en_s \in EN, \exists UE(en_s)\, describe\ en_s\ in\ UDM(Q)$

Where *UE(..)* is a unified entity module in *UDM(Q)*.

IV. Conditional Categorical Activity-Based Model (cCABM)

In this section, we will introduce our proposed model for describing a user experience which includes behavior and emotions experiences. Our proposed model called Conditional Categorical Activity-Based Model, *cCABM*. This proposed model uses category theory elements [22] and activity theory [23, 24] to describe most of user experience elements. The main objective of cCABM is to achieve specific user experience goals and build an optimal model for representing his experience including behavior and emotions in efficient and helpful way.

According to activity theory [24], human behavior can be described using a hierarchical structure consists of three levels: *Activity*, *Action*, and *Operation*. On the contrary, we use category theory [22] to describe all of three levels as *Activity category*, *Action category*, and *Operation category* as shown in Fig. 1.The lowest level, Operation category, represents the basic object routine tasks a user performs. By performing a series of operation categories, the user subject fulfills the action category consists of this series of operation categories, and the hierarchy transfers to the second level, action category. At this point, the short-term goal attached to the action category is also achieved. Achieving a sequence of action categories fulfills the main activity category and hence, the attached main objective is also achieved. So, the overall process of achieving a longer-term goal starts by performing operation categories.

Also, the existence of tasks is based on satisfaction of its related system logical and subject emotional conditions. In other words, both condition types are the catalysts or triggers of tasks. Based on activity theory [24], each task is connected to a set of related system logical conditions. Once the set of these conditions are satisfied the attached task starts. On the contrary, we consider that each task is connected to a set of related system logical conditions and subject emotional conditions. Once the set of both condition types are satisfied the attached task starts. So, these conditions are considered as the triggers of the overall process of achieving an activity category. In addition, we use category theory elements to represent the user emotions as shown in Fig. 2.

A. Proposed Category Activity-Based Model

In our model, we use category theory elements to represents activity theory entities of all three levels. Here, activity entity may be an operation, an action, or an activity as follows.

1) *Operation category*

In our proposed model, an operation transfers to another operation through predescribed tasks given by an operation description. Here, the existence of any operation is based on satisfaction of its related conditions. So, we introduce the following rule:

Rule 1: *for any operation op, there exists a set of conditions such that this operation op exists if and only if these set of all related conditions are satisfied*.

This rule describes the relationship between an operation and its attached set of conditions where if one of these conditions cannot be satisfied, so this operation do not exist. In this paper, we call this relationship as *operation-conditions dependency*.

In some situations, the operation can be existed if some of its attached conditions are satisfied. Therefore, we will classify the operation-conditions dependency into to types of dependent as follows.

Based on category theory [23], we can consider that each operation is a category and we denote as *OPR-Cat*. By using category theory elements: *Objects*, *morphisms*, and *functors*, we can describe elements of OPRC category as follows.

(a) Operation category objects:

Based on the definition of operation, *an operation is a routine task performed by a subject*, the objects of an operation are the elements of that routine task. Let us call these routine elements as atomic steps. For example, If the operation is sending an e-mail message then the objects might be opening a web browser, navigating the e-mail web page, logging into the e-mail account, finding the new message command, clicking new, filling in the recipient e-mail, typing in the subject field, typing in the message body, attaching files (if needed), or clicking send.

(b) Operation category morphisms:

Morphism in operation category can be defined as the way that a routine step (operation object) transfers to the next step. Here, we will define the operation identity morphism, composition and associativity axioms.

Firstly, the operation identity morphism is defined for each object and we can define it by adding a prefix "*re*" to each step. For example, reclicking new, retyping the subject, or resending. So, we will define the identity morphism as follows.

Secondly, the composition operation *oo* and associativity axioms are direct. So, we just need to write them in the symbols form as follows. *for any three tasks objects, A, B, and C, if there is an operation morphism from A to B and another operation morphism from B to C, then there is an operation morphism from A to C.*

Finally, the associativity axiom is satisfied *if for any three morphisms, f, g, and h, there is (f oo g) oo h = f oo (g oo h) such that* $f{:}ts_1 \rightarrow ts_2$, $g{:}ts_2 \rightarrow ts3$ *and* $h{:}ts_3 \rightarrow ts_4$ *where* $ts_1,ts_2,ts_3,ts_4 \in TS(op_k)$ *and* op_k *is an operation category.*

(c) Functors between operations:

In our model, we assume that there is no relationship between individual steps(tasks) in an operation and steps(tasks) in another operation. On the other hand, there is a relationship between two successive operations as a whole called *nextOperation*. For example, "*setting up an appointment with a professor*" operation, then "*face-to-face meeting*" operation, in a context of research environment.

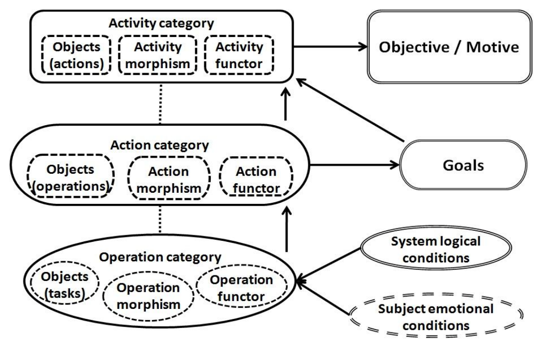

Fig. 1. Architecture of the activity-category principle of hierarchical structure.

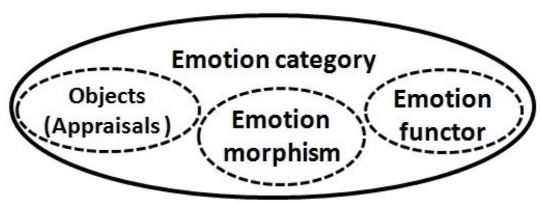

Fig. 2. Subject emotional condition as a category.

2) *Action category*

Each action can be represented as a category. An action transfers to another action through a satisfied set of their related operations based on each action description. Based on category theory [23], we can consider that each action is a category and we denote as *ACT-Cat*. By using category theory elements: *Objects*, *morphisms*, and *functors*, we can describe elements of ACT-Cat category as follows.

(a) Action category objects:

Based on the definition of Action: an action is a set of related operations performed by a subject. Hence, the objects of an action are its related operations.

(b) Action category morphisms:

An *action morphism* can be defined as the way an operation object transfers to the next operation object. Here, we will define the action identity morphism, composition and associativity axioms.

Firstly, the *action identity morphism* is defined for each operation object as the way an operation object transfers to itself such that it must start from the first task of its description.

Secondly, the composition operation, *on*, is defined as follows: *for any three operation objects, A, B, and C, if there is an action morphism from A to B and another action morphism from B to C, then there is an action morphism from A to C.*

Finally, the associativity axiom is satisfied *if for any three morphisms, f, g, and h, there is (f on g) on h = f on (g on h) such that* $f{:}op_1 \rightarrow op_2$, $g{:}op_2 \rightarrow op_3$ *and* $h{:}op_3 \rightarrow op_4$ *where* $op_1,op_2,op_3,op_4 \in ac_j$ *and* ac_j *is an action category.*

(c) Functors between Actions:

In our model, we assume that there is no relationship between individual operations in an action and operations in another action. On the other hand, there is a relationship between two successive actions as a whole called *nextAction*.

3) *Activity category*

Each activity can be represented as a category. Based on category theory [23], we can consider that each activity is a category and we denote as *AT-Cat*. By using category theory elements: *Objects*, *morphisms*, and *functors*, we can describe elements of AT-Cat category as follows.

(a) Activity category objects:

Based on the definition of activity: an activity is a set of related actions performed by a subject. Hence, the objects of an Activity are its related actions. %Let us call these routine elements as atomic steps. For example, If the operation is sending an e-mail message, then, the objects might be opening a web browser, navigating to the e-mail web page, logging into the e-mail account, finding the new message command, clicking new, filling in the recipient e-mail, typing in the subject field, typing in the message body, attaching files (if needed), clicking send.

(b) Activity category morphisms:

An *activity morphism* can be defined as the way an action object transfers to the next action object. Here, we will define the Activity identity morphism, composition and associativity axioms.

Firstly, the *activity identity morphism* is defined for each action object as the way an action object transfers to itself such that it is identity morphism action.

Secondly, the composition operation, *ov*, is defined as follows: *for any three action objects, A, B, and C, if there is an Activity morphism from A to B and another Activity morphism from B to C, then there is an activity morphism from A to C.*

Finally, the associativity axiom is satisfied *if for any three activity morphisms, f, g, and h, there is (f ov g) ov h = f ov (g ov h) such that* $f{:}ac_1 \rightarrow ac_2$, $g{:}ac_2 \rightarrow ac_3$ *and* $h{:}ac_3 \rightarrow ac_4$ *where* $ac_1,ac_2,ac_3,ac_4 \in av_i$ *and* av_i *is an activity category.*

(c) Functors between Activities:

In our model, we assume that there is no relationship between individual actions in an activity and actions in another activity.

4) *Emotion category*

Also, we will use category theory elements to represent a user emotional conditions as follows.

(a) Emotion category objects:

In this model, we will define a user emotion ,en_s, as a set of appraisal objects as [23]. We denote for this set of objects as $AP(en_s)=\{po_t{:}1\leq t \leq T\}$ where T is the total number of its appraisal objects.

(b) Emotion category morphisms:

An *emotion morphism* can be defined as the way an appraisal object transfers to another appraisal object. Here, we will define the emotion identity morphism, composition and associativity axioms.

Firstly, the *emotion identity morphism* is defined for each appraisal object *po* as the way an appraisal object transfers to itself.

Secondly, the composition operation, *oe*, is defined as follows: *for any three appraisal objects, A, B, and C, if there is an emotion morphism from A to B and another emotion morphism from B to C, then there is an emotion morphism from A to C.*

Finally, the associativity axiom is satisfied *if for any three emotion morphisms, f, g, and h, there is (f oe g) oe h*

= f oe (g oe h) such that $f{:}em_1 \rightarrow em_2$, $g{:}em_2 \rightarrow em_3$ *and* $h{:}em_3 \rightarrow em_4$ *where* $em_1, em_2, em_3, em_4 \in en_i$ *and* en_i *is an emotion category.*

(c) Functors between Emotions:
In our model, we assume that there is no relationship between individual appraisals in an emotion and appraisals in another emotion.

The architecture of the proposed category activity-based model is shown in Fig. 3.

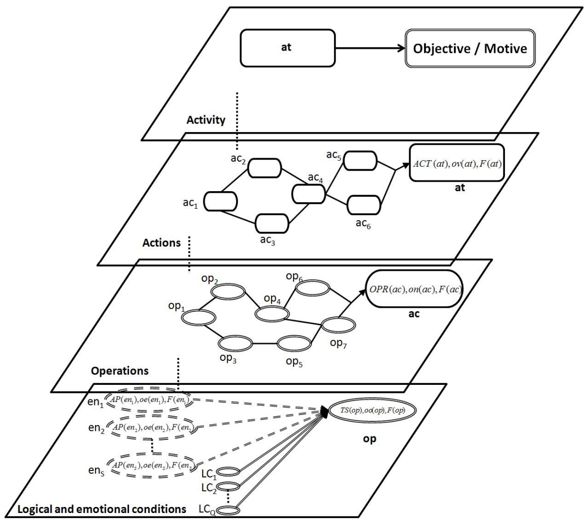

Fig. 3. The architecture of the proposed category activity-based model

V. Case Example and Discussion

In this section, we will discuss our proposed, cCABM, model by showing a creating research paper scenario, *CRP*, as an our case to evaluate this proposed method and proof that cCABM can be used to model a user behavior and a user emotion experiences in efficient and sufficient way. In the remaining of this section, we will describe a research paper scenario, then we will represent this scenario and its related actions, operations, and conditions by using our cCABM model.

Consider a researcher that want to create a new researcher paper. To satisfy his objective, he will set a group of short goals for example: 1) Getting last related research work, 2) Work on the identified problem, 3) realizing the existing solutions, 4) Determining the advantages and disadvantages of the existing solutions, 5) Improving the performance of system according to its problem, and 6) Setting a paper in an academic format. So, satisfying each goal of them, he must do a real set of operations. Each operation will be completed after executing a group of tasks. For example, he will do a searching operation by using books and technical tools as Google search engine to find most of related research work for his research problem. However, if he cannot find a book or cannot use the technical tools, he will not be able to execute his searching operation. These type of conditions represent a logical conditions on a searching operation. In addition, if a search tool cannot help him to get any result, he will get bored or unhappy. So, his operation cannot be started at all. These types of additional conditions represent the emotional conditions on a searching operation. In the other hand, if both condition types exist and satisfied, his operation will start directly.

As shown in our scenario description, there are a lot of actions and short goals which will be satisfied or un-satisfied based on existing completed operations which in turn are based on logical and emotional conditions. In this scenario, these actions, operations, and conditions represent a user experience for this proposed scenario. So, in the reset of this section, we will use our proposed cCABM model to describe this scenario by defining its activity category, all action categories, all operation categories, and emotional condition categories. In addition, we will describe all their related objects and morphisms based on category and activity theories description.

A. *Activity Categories in CRP Scenario*

In our CRP scenario, the creating of a research paper can be considered as the only activity in the system. By using cCABM, we will represent this activity as a category which includes objects, morphisms, and functors. This CRP activity category is described as follows.

1) *Objects of CRP activity*: Based on cCABM, the objects of CRP activity will be a set of actions. These action objects are *Initial reading*, *Identifying a problem*, *Formulating a problem*, *Reading existing solutions*, Proposing a solution, *Validating a solution*, *Writing a paper*, *Publishing a paper* as shown in Table I.

2) Morphisms of CRP activity: Based on objects of CRP activity, the CRP activity morphism means that the transferring from one action to the next action. For example, we can transfer from *Initial reading* to *Identifying a problem* if the short term goal of *Initial reading* action is satisfied. By using this morphism, we will define the identity morphism, the composition rule, and the associativity axioms for CRP activity as follows.

Table I: Operations, actions, and short term goals for CRP scenario

Action	Operations(action objects)	Action short term goals
Initial reading	* Searching * Collecting * Filtering/Selecting * Sorting reading	# Getting last related research work

Identifying a problem	*Describing a problem environment	# Work on the identified problem
Formulating a problem	* Describing a problem symbols * Describing all conditions by using defined parameters * Defining an objective function	# Determining its parameters and parameters and their meanings its formula
Reading existing solutions	* Reading in depth * Creating comparative study	# Realizing the existing solutions # Determining the advantages of the existing solutions # Determining the disadvantages of the existing solutions
Proposing a solution	* Describing a new solution * Formulating a proposed solution	# Treating disadvantages of the existing solutions # Improving the performance of system according to its problem
Validating a solution	* Creating a simulation experiments * Getting a results * Comparing with existing works	# Verifying a solution efficiency and its logic
Writing a paper	* Initial writing * Writing a content * Reviewing a paper	# Setting a paper in an academic format
Publishing a paper	* Starting first submission * Replying a reviewers * Submitting a final version	# Presenting his research work to related community

i) The identity morphism of CRP activity category means that the action object can transfer to itself again. This identity morphism in CRP activity exists if the short term goal of the action did not satisfied at all. For example, if the short term goal of *Initial reading* action did not satisfied, the user must repeat all of its operations again, in other words, the action will return to itself again.

ii) The composition rule of CRP activity category means that if there are three actions as *Initial reading*, *Identifying a problem*, and *Formulating a problem* and there is a morphism from *Initial reading* to *Identifying a problem* and there is a morphism from *Identifying a problem* to *Formulating a problem*. Then, there is a morphism from *Initial reading* to *Formulating a problem*. As shown in CRP scenario this composition is satisfied for all actions in CRP activity.

iii) The associativity axioms of CRP activity category means that if there are a morphism from *Initial reading* to *Identifying a problem*, a morphism from *Identifying a problem* to *Formulating a problem*, and a morphism from *Formulating a problem* to *Reading existing solutions*. Then, if we compose a morphism from *Initial reading* to *Identifying a problem* with a resulted morphism between a morphism from *Identifying a problem* to *Formulating a problem* and a morphism from *Formulating a problem* to *Reading existing solutions*, we will get the same result if

we compose a resulted morphism between a morphism from *Initial reading* to *Identifying a problem* and a morphism from *Identifying a problem* to *Formulating a problem* with a morphism from *Formulating a problem* to *Reading existing solutions*. Also, this associativity axioms of CRP activity category is satisfied for all activity morphisms in CRP scenario. All actions and their related goals is shown in Table I.

3) *Functors between CRP activities*: Our CRP scenario has only one activity category. So, there is no any functor in this CRP scenario.

B. *Action Categories in CRP Scenario*

In our CRP scenario, the CRP activity consists of a set of actions. By using cCABM, we will represent each action as a category which includes objects, morphisms, and functors. Each CRP action category is described as follows.

1) *Objects of CRP action*: Based on cCABM, the objects of CRP action will be a set of operations. For example, the objects of *Initial reading* action are *Searching*, *Collecting*, *Filtering/Selecting*. Table 1, shows all CRP actions and there related operation objects.
2) *Morphisms of CRP action*: Based on objects of CRP action, the CRP action morphism means that the transferring from one operation to the next operation. For example, for *Initial reading* action, we can transfer from *Searching* to *Collecting* if all related tasks of *Searching* operation are completed. By using this morphism, we will define the identity morphism, the composition rule, and the associativity axioms for CRP action as follows.

Table II: Part 1: Examples of tasks of all operations for CRP scenario

Operations	**Tasks(operation objects)**
Searching	t1: determining keywords t2: navigating to digital lib website t3: typing in searching text box t4: pressing a search button t5: finding a results
Collecting	t1: creating a folder t2: downloading a results pdfs files t3: storing them in creating folder

	t3: classifying them
Filtering/Selecting	t1: reading quickly t2: determining a related papers t3: archiving the selected papers
Sorting reading	t1: sorting selected papers based on a defined metric t2: starting reading one by one
Describing a problem environment	t1: specifying a context objects environment t2: describing objects properties t3: describing a relationships t4: describing all required conditions
Describing a problem symbols and their meanings	t1: determining all parameters symbols and their t2: giving a symbol for each meanings parameter t3: describing each symbol meaning
Describing all conditions by using defined parameters	t1 determining all conditions conditions by using parameters defined parameters t2: describing all conditions t3: writing a formula for each condition
Defining an objective function	t1: determining its elements function t2: describing all elements t3: writing all related conditions t4: writing a suitable objective formula

i) The identity morphism of CRP action category means that the operation object can transfer to itself again. This identity morphism in CRP action exists if the execution of all related tasks did not satisfy the goal of this operation object. For example, if the related tasks of *Searching* operation object, as shown in Table 2, did not give a user the required results, the user must repeat all of its tasks again, in other words, the operation will return to itself again.

ii) The composition rule of CRP action category means that if there are three operations as *Searching*, *Collecting*, and *Filtering/Selecting* for *Initial reading* action and there is a morphism from *Searching* to *Collecting* and there is a morphism from *Collecting* to *Filtering/Selecting*. Then, there is a morphism from *Searching* to *Filtering/Selecting*. As shown in CRP scenario this composition is satisfied for all operation objects in CRP actions.

iii) The associativity axioms of CRP action category means that if there are a morphism from *Searching* to *Collecting*, a morphism from *Collecting* to *Filtering/Selecting*, and a morphism from *Filtering/Selecting* to *Sorting reading*. Then, if we compose a morphism from *Searching* to *Collecting* with a resulted morphism between a morphism from *Collecting* to *Filtering/Selecting* and a morphism from *Filtering/Selecting* to *Sorting reading*, we will get the same result if we compose a resulted morphism between a morphism from *Searching* to *Collecting* and a morphism from *Collecting* to *Filtering/Selecting* with a morphism from *Filtering/Selecting* to *Sorting reading*. Also, this associativity axioms of CRP action category is satisfied for all action morphisms in CRP scenario. All actions and their related operation objects are shown in Table 1.

Table III: Part 2: Examples of tasks of all operations for CRP scenario

Operations	Tasks(operation objects)
Reading in depth	t1:sorting based on defined a creitra t2: selecting a paper t3: reading in details t4: finding their solutions t5: finding basic ideas t6: finding advantages and disadvantages
Creating comparative study	t1: determining comparison study metrics t2; describing comparison metrics t3: creating a comparison table t4: concluding the comparison
Describing a new solution	t1: describing a solution goals solution t2: describing basic ideas
Formulating a proposed solution	t1: describing all assumptions solution t2: describing system models t3: formulating system models t4: creating a proposed formula
Creating a simulation experiments	t1: defining simulation experiments parameters t2: determining parameters values
Getting a results	t1: assigning values to current experiment parameters t2: running the simulation t3: recording results
Comparing with existing works	t1: creating result charts works t2: analyzing each chart t3: concluding results

3) *Functors between CRP actions*: Based on cCABM, there is only a relationship between two successive actions as a whole. For example, in CRP scenario there is a relationship between *Initial reading* and *Identifying a problem* and between *Identifying a problem* and *Formulating a problem*. So, there is no any functor between actions in this CRP scenario.

C. Operation Categories in CRP Scenario

In our CRP scenario, each operation consist of a set of tasks as shown in Tables 2, 3, and 4 By using cCABM, we will represent each operation as a category which includes objects, morphisms, and functors. This CRP operation category is described as follows.

1) *Objects of CRP operation*: Based on cCABM, the objects of CRP operation will be a set of tasks. For example, the

object tasks of *Searching* operation are *determining keywords*, *navigating to digital lib website*, *typing in searching text box*, *pressing a search button*, and *finding a results* as shown in Table 2.

2) *Morphisms of CRP operation*: Based on objects of CRP operation, the CRP operation morphism means that the transferring from one task to the next task. For example, we can transfer from *determining keywords* to *navigating to digital lib website* if the conditions of operation object are satisfied. By using this morphism, we will define the identity morphism, the composition rule, and the associativity axioms for CRP operation as follows.

i) The identity morphism of CRP operation category means that the task object can transfer to itself again. This identity morphism in CRP operation exists if the related condition of this operation object did not satisfied. For example, if digital lib and Internet access do not available through doing *navigating to digital lib website* task, the user must repeat this task when both of these conditions are available, in other words, the task will return to itself again.

ii) The composition operation of CRP operation category means that if there are three tasks as *determining keywords, navigating to digital lib website*, and *typing in searching text box* and there is a morphism from *determining keywords* to *navigating to digital lib website* and there a morphism from *navigating to digital lib website* to *typing in searching text box*. Then, there is a morphism from *determining keywords* to *typing in searching text box*. As shown in CRP scenario this composition is satisfied for all task objects in CRP operations.

iii) The associativity axioms of CRP operation category means that if there are a morphism from *determining keywords* to *navigating to digital lib website*, a morphism from *navigating to digital lib website* to *typing in searching text box*, and a morphism from *typing in searching text box* to *pressing a search button*. Then, if we compose a morphism from *determining keywords* to *navigating to digital lib website* with a resulted morphism between a morphism from *navigating to digital lib website* to *typing in searching text box* and a morphism from *typing in searching text box* to *pressing a search button*, we will get the same result if we compose a resulted morphism between a morphism from *determining keywords* to *navigating to digital lib website* and a morphism from *navigating to digital lib website* to *typing in searching text box* with a morphism from *typing in searching text box* to *pressing a search button*. Also, this associativity axioms of CRP operation category are satisfied for all operation morphisms in CRP scenario.

3) *Functors between CRP operations*: Based on cCABM, there is only a relationship between two successive operations as a whole. For example, in CRP scenario there is a relationship between *Searching* and *Collecting* and between *Collecting* and *Filtering/Selecting*. So, there is no any functor between operations in this CRP scenario.

Table IV: logical and emotional conditions for searching and reviewing paper operations in CRP scenario

Operation	**Logical condition**	**Emotional condition**
Searching	lc1: Availability of digital lib lc2: Availability of Internet access	en1: Joy en2:Satisfaction en3: Happiness en4: Anger en5: Boring en6: Tiredness en7: Failure en8: Unsatisfaction
Reviewing paper	lc1:Completed content lc2:Readable paper lc3: Available time before deadline	en1: Joy en2: Satisfaction en3: Happiness en4: Anger en5: Boring en6: Tiredness en7: Failure en8: Unsatisfaction

D.Logical and Emotional Conditions in CRP Scenario

In our cCABM, any operation depends on a set of conditions to start its tasks where each condition in this set can be a logical or an emotional condition. The logical condition means that the result of a condition is true or false and it does not depend on the feeling of a user. While the emotional condition represents the degree of a user emotions regards to system operations. Table 5, shows logical and emotional conditions of *Searching* and *Reviewing paper* operations in CRP scenario. In cCABM, we used a representation model which introduced in [25] to describe any emotional condition as a category with its appraisal objects, morphisms, and functors.

In this case example, we used cCABM to describe a user behavior and emotions experiences for CRP scenario and we clarified how to use category and activity theories in representing these user experiences by using a conditional mathematical modeling. cCABM can give an efficient and helpful way for describing a user experience which can be used by researchers, developers, and designers for building a lot of professional, smart, and interactive applications.

VI. CONCLUSION

In this paper, we proposed a conditional mathematical model called conditional categorical activity-based model, cCABM, for describing a user behavior and emotion experiences by using category and activity theories. cCABM can describe any user behavior as an activity that consists of a set of actions which consists of a set of operations where their existence depends on a set of logical and emotional conditions. Also, this paper introduced a creating research paper scenario as a case example for cCABM. This case example validates that cCABM can be used as an efficient and helpful tool for describing a user experience which can be used by researchers, developers, and designers for building a lot of professional, smart, and interactive applications.

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