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Original Article

Modeling Complements of Positive Integer Numbers Using Advanced Radix and Diminished-Radix Methods

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Abstract *- Complement operations have been used to simplify the subtraction and logical not operations. These operations have been solved using the radix and the diminished-radix methods, respectively. These methods did not consider the sign-bit of the given positive integer numbers. To overcome the problem of the existing methods, this paper has been developed to model the complements of positive integer numbers using the advanced radix and diminished-radix methods.*

Keywords *- Number System, Complement, Radix method and Diminished-Radix Method*

I. INTRODUCTION

In a positional number system, each number is represented by a string of digits in which each digit position has an associated weight r_i, where r is the radix or base of the number system. The general form of a number in a number system with radix-r is [1] –[4]:

$$x = (\ldots\ldots.. x_{k-1}x_{k-2}x_{k-3}\ldots x_1x_0 \ldots\ldots.)_r$$
$$= \sum_{i=-\infty}^{\infty} x_i r^i$$

Where the value of any digit x_i is an integer in the range $0 \leq x_i \leq r$. For known digits, the general form of the number is:

$$x = (x_{k-1}x_{k-2}x_{k-3}\ldots x_1x_0)_r$$
$$= \sum_{i=0}^{k-1} x_i r^i$$

For a number with a sign-magnitude digit, the general form of a number in a system with radix-r is represented in Equation (i):

$$x = (x_k x_{k-1}x_{k-2}x_{k-3}\ldots x_1x_0)_r$$
$$= \sum_{i=0}^{k} x_i r^i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

A floating-point (FP) number can be represented in the form $\pm M \times b^e$, where m, called the mantissa, represents the fractional part of the number and is normally represented as a single binary fraction; e represents the exponent and b represents the base radix of the exponents [1]-[4]. The floating-point numbers are represented as:

$$x = (\pm 1) \times m \times 2^{\pm exponent}$$
$$x = sign \times Mantissa \times 2^{\pm exponent}$$

where $1.0 \leq m < 2$ and $m = (b_0.b_1b_2b_3\ldots\ldots.)_2$

To store the number in floating-point representation, a computer word is divided into three fields: sign bit, exponent and mantissa, respectively. In a 32-bits single-precision floating-point representation, the most significant bit is used for the sign (s) bit with 0 for a positive number and 1 for a negative number. The next 8-bits stands for the exponent, and the last 23-bits is used for the mantissa. The 32-bits single-precision floating-point representation is shown in Fig 1.

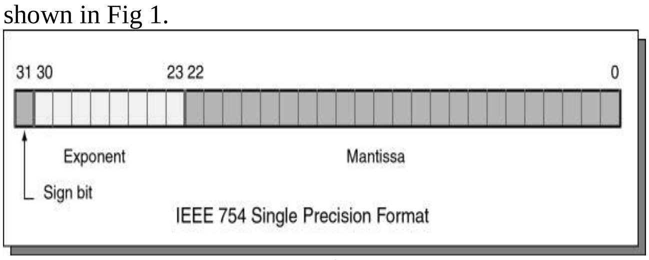

Fig. 1 32-bits single-precision floating-point representation
(Source: [1]-[4])

In a 64-bits double-precision floating-point representation, the most significant bit is used for the sign (s) bit with 0 for a positive number and 1 for a negative number. The next 11-bits are used for the exponent and the last 52-bits for the mantissa. The 64-bits single-precision floating-point representation is shown in Fig 2.

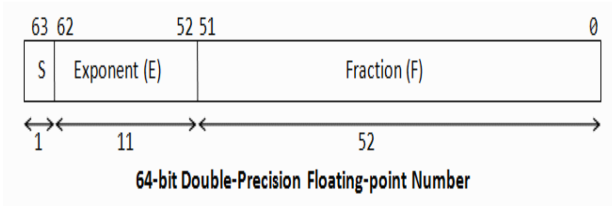

Fig. 2 64-bits double precision floating-point representation
(Source: [2]-[3]).

The representation of the fields is shown in Table 1.

Table 1. Fields size representation of the floating-point numbers *(Source: [1]).*

Floating point type	*Sign field size*	*Exponent field size*	*Mantissa field size*	*Bias*
Single precision 32bits computers	***1***	***8***	***23***	***127***
Double precision 64bits computers	***1***	***11***	***52***	***1023***

Similarly, integer numbers can be represented as a word that is divided into two files: sign-field and the integer number field. An integer number representation of a word is shown in Fig 3.

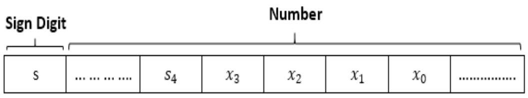

Fig. 3 Integer number representation

Where = x_k is the sign-magnitude digit of the number, the sign digit takes two values: 0 and (r-1). The (r-1) value represents a negative sign whose value is always -1 in any radix -r.

A number system in base or radix-r is a system that uses r-distinct symbols or elements, ranging from 0 to (r-1). For example, the elements of a number system in base 10 are represented in a set. The 10 symbols or elements of the decimal number or radix-10 number system are:

$$S_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Numbers are represented by strings of digit symbols. These number is derived by concatenating one or more digit symbols of a particular base. Generally, any radix number is a sum of the series of power of the base times a number from 0 to 1 less than the base. The syntax of number representation is shown in Equation (ii).

Number =............ num[n]xbase^n + num[n-1]x base^{n-1} +………. + num[0] x base^0 + ……….. (ii)

For example, the string of digits 742.5_{10} is interpreted to represent a quantity expressed in the decimal base as follows:

$$742.5 = 700 + 40 + 2 + \frac{5}{10} = 7x10^2 + 4x10^1 + 2x10^0 + 5x10^{-1}$$

For example, the string of digits 101101 is interpreted to represent a quantity

$$101101 = 1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$$
$$= 32 +0 +8 +4 +0 +1$$
$$= 45$$

Besides the decimal number system, the binary (radix-2), the octal (radix 8) and hexadecimal (radix 16) are important in the digital computer system.

II. REVIEW OF RELATED WORKS

Complements are used in digital computers for simplifying subtraction operations and logical manipulations. There are two types of complement operations for a number system of base r. These are:

- The r's complement operation
- The (r-1)'s complement operation

For instance, when dealing with a binary number system where the value of r is 2, the complements are 2's complement and 1's complement. Similarly, for a decimal system where the value of r is 10, the complements are 10's complement and 9's complement [mano].

A. The Radix Complement Method

[3]-[4] developed a mathematical model for the radix complement method. The radix complements, also called the r's complement, is defined as follows:

If a positive number N is given in base r, with an integer part of n digits, the r's complement of N is shown in Equation (iii).

$$\boldsymbol{r'scomplement} = \begin{cases} \boldsymbol{r^n - N \; for \; N \neq 0} \\ \boldsymbol{0 \; for \; N = 0} \end{cases} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(\boldsymbol{iii})$$

For example, the 9's complement of (23450) is calculated as follows:

$$10's\ complement = (10^5 - 1) - 23450$$
$$= 100000 - 23450$$
$$= 10000 - 23450 = 76550$$

B. Diminished-Radix Complement Method

[3]-[4] also developed a mathematical model for the diminished-radix complement method. The diminished-radix complements, also called the (r-1)'s complement, is defined as follows:

If a positive number N is given in base r with an integer part of n-digits and a fractional part of m-digits, then the (r-1)'s complement of N is shown in Equation (iv).

$$\boldsymbol{(r-1)'s\ complement} = \begin{cases} \boldsymbol{r^n - N - 1 \;\; for \; N \neq 0} \\ \boldsymbol{0 \quad for \; N = 0} \end{cases} \;\ldots\ldots\ldots\ldots\ldots..(\boldsymbol{iv})$$

For example, the 9's complement of (23450) is calculated as follows:

$$9's\ complement = (10^5 - 1) - 23450$$
$$= 100000 - 23450$$
$$= 100000 - 23450 = 76550$$

The r's and (r-1)'s complements are related by the Equation:

$$\boldsymbol{r's\ complement = (r-1)'s\ complement + 1}$$

For example, the 9's complement of (23450) is calculated as follows:

$$9's\ complement = (10^5 - 1) - 23450$$
$$= (100000 - 1) - 23450$$
$$= 99999 - 23450 = 76549$$

These methods do not consider the sign-bit of integer numbers.

III. ADVANCED MODELING APPROACHES

To Introduce The Sign-Bit, The Researcher has developed the advanced radix complement and diminished-radix complement methods.

A. Advanced Radix Complement Method

The advanced radix complement method is defined in Equation (v).

$$\boldsymbol{r's complement} = \begin{cases} \boldsymbol{r^{n+1} - N \ for \ N \neq 0} \\ \boldsymbol{0 \ for \ N = 0} \end{cases} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (\boldsymbol{v})$$

Where r represents the base or radix of the integer number of digit n and (n+1) digit represents the sign-magnitude of the number. The sign-magnitude can take positive or negative values. A negative sign-magnitude of -1 in the (n+1) digit takes a value (r-1), while a positive sign-magnitude takes value 0 in the most significant bit position.

For example, the 10's complement of (23450) is calculated as follows:

$$10's complement = 10^{5+1} - 23450 = 976550$$

This number is translated as:

$$-1 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 + 5 \times 10^1 + 0 \times 10^0$$
$$= -100000 + 70000 + 6000 + 500 + 50 + 0$$
$$= -23450$$

B. Advanced Diminished-Radix Complement Method

The advanced diminished-radix complement method is defined as:

If a positive number N is given in base r with an integer part of n-digits and a fractional part of m-digits, then the (r-1)'s complement of N is shown in Equation (vi).

$$(\boldsymbol{r-1})'\boldsymbol{s \ complement} = \begin{cases} (\boldsymbol{r^{n+1} - 1}) - \boldsymbol{N \ for \ N \neq 0} \\ \boldsymbol{0 \ for \ N = 0} \end{cases} \ldots\ldots\ldots\ldots. (\boldsymbol{vi})$$

The r's and (r-1)'s complements are related by the Equation:

$$\boldsymbol{r's \ complement} = (\boldsymbol{r-1})'\boldsymbol{s \ complement} + \boldsymbol{1} \qquad (\boldsymbol{vii})$$

For example, the 9's complement of (23450) is calculated as follows:

$$9's \ complement = (10^{5+1} - 1) - 23450$$
$$= (1000000 - 1) - 23450$$
$$= 999999 - 23450 = 976549$$

This number is translated as:

$$-1 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 + 4 \times 10^1 + 9 \times 10^0$$
$$= -100000 + 70000 + 6000 + 500 + 40 + 9$$
$$= -23451$$

Therefore, the r's complement is calculated as:

r's complement = -23451 +1 = -23450

IV. DISCUSSION OF RESULTS

From the results obtained, it can be inferred that the advanced methods developed or modelled by the researcher in this paper have been able to handle the sign-bit and sign-magnitude of the positive integer numbers used in the number systems.

V. CONCLUSION

The researcher has modelled or developed mathematical models for simplifying the subtraction and logical not operations in a digital computer putting the sign-magnitude and the sign-bit into consideration.

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