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Original Article

Cayley Graph on Nilpotent Groups with and without Hamilton Path

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Abstract - *A Cayley graph with a hamilton path as finite when every vertices are connected and do not contain a Hamilton path when it is infinite has been constructed. Cayley graph must be directed and must contain nilpotent, commutator subgraph.*

Keywords - *Cayley graph, hamilton path, nilpotent group, commutator subgraph.*

I. INTRODUCTION

Let G be a group. Where any subset of S of the finite group, the Cayley digraph is the directed graph whose vertices are the elements of G and with a directed edge $g \longrightarrow gs$ for every $g \longrightarrow G$ and $s \in S$, denoted $Cay^{\rightarrow}(G; S)$ [1].

Every connected Cayley graph has a hamilton path if G has a prime-power order; otherwise, not [3] as this is not executable for many directed edges. However, only in infinite groups that are not solvable [4].

Proposition 1.1 In many infinite groups, G, where every connected Cayley graph on G has a Hamilton cycle and G is not solvable [4].

Here, alternating group A_5 *(order 60)* is a simple nonabelian group.

Proposition 1.2 $P \equiv 1 \ (mod\ 30)$ where P = *prime number*, then every connected Cayley graph on the direct product $A_5 \times \mathbb{Z}_p$ has a hamilton cycle [4].

Remark. The assumption $G = P \times A$ in Proposition 1.2 is equivalent to assuming that G is nilpotent and the commutator subgroup of G has a prime-power order [1].

Proposition 1.3 On any nilpotent group containing connected Cayley digraph of out valence 2 has a hamilton path [1].

Corollary 1.3 On any nilpotent group containing connected Cayley digraph of out valence ≤ 4 has a hamilton path [1].

Remark. If $S = \{a,b\}$ is a 2-element generating set of a group G where $|a| = 2,\ |b| = 3,\ |G| > 9|ab^2|$, then $Cay^{\rightarrow}(G\ ;a,b)$ does not have a hamilton path [1].

Proposition 1.4 Let G be an abelian group and let $a,b,k \in G$ such that k be an element of order 2. G is cyclic if the Cayley digraph$^{\rightarrow}(G;\ a,b,\ b+k)$ is connected but does not have a hamilton cycle [3].

II. NOTATION

Notation 2.1 Let G be a group and S be a subset of G.

- *Cay(G:S)* denotes the Cayley graph of G
- concerning S, where the vertices are the elements of G and an edge joining g to gs for every $g \rightarrow G$ and $s \in S$.
- $G' = [G,\ G]$ denotes the commutator subgraph of G.
- $S^T = \{ S^T | s \in, S\}$ for any $r \varepsilon \mathbb{Z}$.
  $S^{\pm 1} = S \cup S^{-1}$
- $\# S$ is the cardinality of S

Notation 2.2

- G is a nilpotent finite group
- N is a normal cyclic subgroup of G that contains G
- $g \rightarrow \bar{g}$ is a homomorphism from G to $G/N = G.'$
  $S = \{ \sigma_1, \sigma_2, \ldots\ldots, \sigma_l \}$ is a subset of G Where $l = \neq S = \neq \bar{S} \geq 2$ [2].

III. PROOF OF THE PROPOSITIONS

Proof of proposition 1.2 Let $Cay^{\rightarrow}(G;\ a,b)$ be a connected where $G = P \times A$ and if $H = (S^{-1} S)$ be the arc-forcing subgraph assuming S is minimal.

Case 1. $H \neq G$ has been assumed on $|G|$ where on the Cayley graph has a Hamilton path H in $Cay^{\rightarrow}(G;S)$.

Case 2. $H = G$, $G = H = (a^{-1}S - \{e\})$ where the subset of S_0 of S ,then S is minimal and such that $(\bar{S_0})$ = P.Since $G/P \cong A$ is a abelian this implies $[\ G,\ G\ ] \subseteq$

(S_0). If $N \neq G$, assume *[G, G]* is non-trivial and otherwise provides a hamilton path [1]. Let $(S_i)^n_{i=1}$ be a Hamilton path in $Cay^{\rightarrow}(N; S_0)$.

Proof of proposition 1.3 Let {a,b} be a 2-element generating set for *G* . Then the arc-forcing subgroup $H = (a^{-1}b)$ is cyclic, so it is abelian and provides a hamilton path in $Cay^{\rightarrow}(G;a,b)$*[1]*

Proof of Corollary 1.3 Let *Cay (G; S)* is connected with valence ≤ 4 when *G* is nilpotent [1]. S_2 has a set of elements of order 2 in *S* and *P* be the 2-subgraph of $G = P \times K$, where | *K* | is odd.

Requiring $\neq S - \neq S_2 \leq 1$ since $S_2 \subseteq P$ as $K \cong G/P$ is cyclic. Therefore proposition 1.2 applies $\neq S - \neq S_2 \leq 2$ then $4 \geq$ *valence of* $Cay(G; S) = \neq (S \cup S^{-1}) = 2 (\neq S - \neq S_2) + \neq S_2 \geq 2 (\neq S - \neq S_2) \geq 2.2$. So $\neq S = 2$ *(and* $S_2 = \emptyset$*)*

Proof of proposition 1.4 By assuming *G* is not cyclic, and it has been shown that the Cayley digraph has a Hamilton cycle if it is connected.

Constructing a subdigraph H_0 of *G* where *G* is in the place of $\mathbb{Z}_{2k}$ with $|G|$ in the place of *2k* and with $|an|$ in the place of *d* (when $k \notin (a)$ and $k \in (a)$).

Every vertex of H_0 has both invalence 1 and outvalence 1. $(a\text{-}b) \neq G$ as *G* is not cyclic implies that *(a-b)* has even order. So, $a= a'+k'$ and $b=b'+k'$ for $a',b' \in (a\text{-}b)$ and $k',k'' \in (k)$ as can be shown that $k' = k''$*[4]*.

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