Hybrid impulsive and switching control for the four-dimensional energy resources supply-demand system

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***Abstract*—In this paper, a theoretical method for analyzing the stability of the energy supply-demand system under the impulsive and switching control is considered. By employing the theory of impulsive differential equation, several sufficient conditions ensuring the exponential stability of the system are obtained. Numerical simulations are given to verify the effectiveness of the theoretical analysis.**

***Keywords*—Energy, supply-demand, hybrid control, switched Lyapunov function.**

I. Introduction

Energy supply-demand security is the core of energy security, so ensuring energy supply-demand security is not only to solve the most important prerequisite for energy security, but also has a crucial role for the energy and economic development. By analyzing the energy resources demand in the eastern regions of China and the energy resources development in the western regions of China, Sun ect[1] have established a continuous four-dimensional non-linear differential system to analyze the dynamical behavior of the system.

The study on switching systems as a special hybrid system has attracted great attention since 1990s [2]. A switching system can be considered as a class of hybrid dynamical systems consisting of a family of continuous or discrete time subsystems with a logical rule that orchestrates the switching between them [3]. Switching among different controllers even for a single process can be viewed as a switching system. This area of research has many practical applications in fields such as applied mathematics, engineering and computer science [4-7]. Meanwhile, many practical systems in physics, biology, engineering, and information science exhibit impulsive dynamical behaviors due to abrupt changes at certain instants during the dynamical process [8-10]. The impulsive and switching control is a widely used control strategy in some biological systems particularly such as biological neural networks and bursting rhythm models in pathology [11-12].

Date up to now, there are no reports in present literature on impulsive and switching control for energy supply-demand system. Inspired by the above discussion, the main purpose of this paper is to investigate impulsive and switching control for the energy supply-demand system. By using the theory of impulsive differential equations, several sufficient conditions are obtained to ensure the exponential stability of the system. Finally, the numerical example demonstrates the effectiveness of the proposed schemes.

This paper is organized as follows. In Section 2, model description is introduced. In Section 3, problem formulation and some preliminaries are given. In Section 4, the exponential stability of the energy supply-demand system is studied and some sufficient conditions is derived. In Section 5, a numerical example is given to show the effectiveness of the obtained results. Finally, Conclusions are drawn in Section 6.

II. Model description

Sun ect[1] have established a continuous four-dimensional non-linear differential system to analyze the dynamical behavior of the system. The four-dimensional energy supply-demand system is described by the following:

$$\begin{cases} \dfrac{dx}{dt} = a_1 x(1-\dfrac{x}{M}) - a_2(y+z) - d_3 u, \\ \dfrac{dy}{dt} = -b_1 y - b_2 z + b_3 x[N-(x-z)], \\ \dfrac{dz}{dt} = c_1 z(c_2 x - c_3), \\ \dfrac{du}{dt} = d_1 x - d_2 u. \end{cases} \tag{1}$$

Where $x(t)$ is A region's energy resources demand, $y(t)$ is B region's energy resource supply to A region. $z(t)$ is the energy resource import in A region. $u(t)$ is the renewable energy resources in A region. a_i, b_i, c_i, d_i, M, N are positive constants. When the system's parameters are chosen as follows:

$a_1 = 0.09$, $a_2 = 0.15$, $b_1 = 0.06$, $b_2 = 0.082$, $b_3 = 0.07$, $c_1 = 0.2$, $c_2 = 0.5$, $c_3 = 0.4$, $d_1 = 0.1$, $d_2 = 0.06$, $d_3 = 0.08$, $M = 1.8$, $N = 1$.

We can obtain three equilibrium points: $O(0,0,0,0)$, $S_1(1.75, -1.52, 0, 2.91)$ and $S_2(0.8, 0.669, -1.11, 1.33)$, which are unstable. Let initial condition $(0.82, 0.29, 0.48, 0.1)$ and parameters are fixed as above, a chaotic attractor is observed. A four-dimensional energy resources chaotic attractor is shown in Fig1.

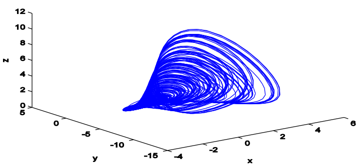

Fig.1. A four-dimensional energy resources chaotic attractor: $3D$ view $(x-y-z)$.

III. PROBLEM FORMULATION AND SOME PRELIMINARIES

Let (x^*, y^*, z^*, u^*) be the equilibrium point of system (1), by using the following transformation: $x_1 = x - x^*$, $x_2 = y - y^*$, $x_3 = z - z^*$, $x_4 = u - u^*$, the equilibrium point (x^*, y^*, z^*, u^*) can be shifted to the origin:

$$\dot{\bar{x}} = A\bar{x} + f(\bar{x}), \qquad (2)$$

where

$\bar{x}^T = [x_1, x_2, x_3, x_4]$,

$f^T(\bar{x}) = [-\frac{a_1}{M}x_1^2, -b_3x_1^2 + b_3x_1x_3, c_1c_2x_1x_3, 0]$,

$$A = \begin{bmatrix} a_1(1-\frac{2x^*}{M}) & -a_2 & -a_2 & -d_3 \\ b_3(N+z^*-2x^*) & -b_1 & b_3x^*-b_2 & 0 \\ c_1c_2z^* & 0 & c_1(c_2x^*-c_3) & 0 \\ d_1 & 0 & 0 & -d_2 \end{bmatrix},$$

the nonlinear system (2) with the control input can be described as

$$\dot{\bar{x}} = A\bar{x} + f(\bar{x}) + u(t,\bar{x}),$$

where $u(t,\bar{x})$ is the control input. We can construct a hybrid impulsive and switching controller $u = u_1 + u_2$ for (2) as follows:

$$u_1(t) = \sum_{k=1}^{\infty} B_{1k}\bar{x}(t)l_k(t), \quad u_2(t) = \sum_{k=1}^{\infty} B_{2k}\bar{x}(t)\delta(t-t_k^-), \qquad (3)$$

where B_{1k} and B_{2k} are 4×4 constant matrices, $\delta(\cdot)$ is the Dirac impulse. And $l_k(t)=1$ as $t_{k-1} \le t < t_k$, otherwise $l_k(t)=0$ with discontinuity points, $t_1 < t_2 < \cdots t_k \cdots$, $\lim_{k\to\infty} t_k = \infty$, $t_0 \ge 0$ is the initial time.

From (3), $u_1(t) = B_{1k}\bar{x}(t)$, $t \in [t_{k-1}, t_k)$, $k = 1,2,...$, this implies that the controller $u_1(t)$ switches its values at every instant t_k, and without loss of generality, it is assumed that $\bar{x}(t_k) = \bar{x}(t_k^-) = \lim_{h\to 0^+} \bar{x}(t_k - h)$. On the other hand, $u_2(t) = 0$ as $t \ne t_k$, and

$$\bar{x}(t_k) - \bar{x}(t_k - h) = \int_{t_k-h}^{t_k} [A\bar{x} + f(\bar{x}) + u_1(s) + u_2(s)]ds,$$

where $h > 0$ is sufficiently small. As $h \to 0^+$, this reduces to $\Delta\bar{x}(t_k) = \bar{x}(t_k) - \bar{x}(t_k^-) = B_{2k}\bar{x}(t_k^-)$, where $\bar{x}(t_k^-) = \lim_{h\to 0^+}\bar{x}(t_k - h)$. This implies that the controller $u_2(t)$ has the effect of suddenly changing the state of (2) at the points t_k. Therefore, $u_2(t)$ is an impulsive controller, and $u_1(t)$ is a switching controller.

Accordingly, with the hybrid impulsive and switching control (3), the nonlinear system of (2) becomes a nonlinear hybrid impulsive and switching system

$$\begin{cases} \dot{\bar{x}} = A\bar{x} + f(\bar{x}) + B_{1k}\bar{x}, & t \in [t_{k-1}, t_k), \\ \Delta\bar{x} = B_{2k}\bar{x}(t_k^-), & t = t_k, \\ \bar{x}(t_0) = \bar{x}_0, & k = 1,2... \end{cases} \qquad (4)$$

We can rewrite (4) in the form of

$$\begin{cases} \dot{\bar{x}} = A_{i_k}\bar{x} + f(\bar{x}), & t \in [t_{k-1}, t_k), \\ \Delta\bar{x} = B_{2k}\bar{x}(t_k^-), & t = t_k, \\ \bar{x}(t_0) = \bar{x}_0, & k = 1,2... \end{cases} \qquad (5)$$

Where $A_{i_k} = A + B_{1k}$. The switching signal $\sigma : R_+ \to \{1,2,....m\}$, which is represented by $\{i_k\}$ according to $[t_{k-1}, t_k) \to i_k \in \{1,2,....m\}$, is a piecewise constant function. Obviously, system (5) has m different modes.

Assumption 1. Considering the actual meaning of model (1), $x(t)$, $y(t)$, $z(t)$, $u(t)$ are bounded, so let $\bar{x}^T\bar{x} \le L$, where L is positive constant, $\bar{x} = [x_1, x_2, x_3, x_4]^T$.

Lemma 1. If $P \in R^{n\times n}$ is a symmetric and positive definite matrix, $Q \in R^{n\times n}$ is asymmetric matrix, then $\lambda_{\min}(P^{-1}Q)\bar{x}^TP\bar{x} \le \bar{x}^TQ\bar{x} \le \lambda_{\max}(P^{-1}Q)\bar{x}^TP\bar{x}$, $\bar{x} \in R^n$.

IV. MAIN RESULTS

Theorem. Assume that Assumption 1 holds, there exist symmetric and positive definite matrices P_{ik}, $\alpha > 0$ is a constant, and the nonlinear impulsive and switching system (5) satisfies

$$\sum_{i=1}^{k-1}\ln(\rho\beta_i) + \sum_{i=1}^{k-1} b_i(t_i - t_{i-1}) + b_k(t - t_{k-1}) \le \psi(t_0, t) \le -\alpha(t - t_0),$$

$$t \in [t_{k-1}, t_k). \qquad (6)$$

$$\lambda_{\max}\left[(I + B_{2k})^T(I + B_{2k})\right] \le \beta_k, \quad k = 1,2... \qquad (7)$$

where

$$b_k = \left\{\lambda_{\max}\left[P_{i_k}^{-1}\left(A_{i_k}^TP_{i_k} + P_{i_k}A_{i_k}\right)\right] + \lambda_{\max}\left(P_{i_k}^{-1}P_{i_k}^{\ T}P_{i_k}\right) + \frac{L\left(\frac{a_1^2}{M^2} + \frac{9}{4}b_3^2 + \frac{c_1^2c_2^2}{4}\right)}{\lambda_{\min}(P_{i_k})}\right\},$$

$$\rho = \max_{1\le i_k\le m}\{\rho_{i_k}^2\}, \quad \rho_{i_k} = \sqrt{\lambda_{\max}(P_{i_k}) / \lambda_{\min}(P_{i_k})}, \quad i_k \in \{1,2,...,m\},$$

$\lambda_{\max}(\cdot)$, $\lambda_{\min}(\cdot)$ donate the maximum and minimum eigenvalues of the matrix, respectively. Then the origin of the nonlinear impulsive and switching system (5) is exponentially stable.

Proof. Construct the switched Lyapunov function in the form of

$$V_{i_k} = \bar{x}^TP_{i_k}\bar{x}, \; i_k \in \{1,2,...,m\}. \qquad (8)$$

The total derivative of V_{i_k}, with respect to (5), is

$$\dot{V}_{i_k} = \dot{\bar{x}}^T P_{i_k} \bar{x} + \bar{x}^T P_{i_k} \dot{\bar{x}} = \bar{x}^T \left(A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} \right) \bar{x} + 2 f^T P_{i_k} \bar{x}.$$

Notice that

$$2 f^T P_{i_k} \bar{x} \le f^T f + \bar{x}^T P_{i_k}^T P_{i_k} \bar{x},$$

$$f(\bar{x}) = [-\frac{a_1}{M} x_1^2, -b_3 x_1^2 + b_3 x_1 x_3, c_1 c_2 x_1 x_3, 0]^T.$$

Let

$$g_1 = -\frac{a_1}{M} x_1^2,\quad g_2 = -b_3 x_1^2 + b_3 x_1 x_3,\quad g_3 = c_1 c_2 x_1 x_3,\quad g_4 = 0.$$

We have

$$\frac{|g_1|}{x_1^2 + x_2^2 + x_3^2 + x_4^2} \le \frac{\frac{a_1}{M} x_1^2}{x_1^2 + x_3^2} \le \frac{a_1}{M},$$

$$\frac{|g_2|}{x_1^2 + x_2^2 + x_3^2 + x_4^2} \le \frac{b_3 |x_1 x_3 - x_1^2|}{x_1^2 + x_3^2} \le \frac{b_3 \left(|x_1 x_3| + x_1^2 \right)}{x_1^2 + x_3^2} \le \frac{3}{2} b_3,$$

$$\frac{|g_3|}{x_1^2 + x_2^2 + x_3^2 + x_4^2} \le \frac{c_1 c_2 |x_1 x_3|}{x_1^2 + x_3^2} \le \frac{c_1 c_2}{2},$$

$$|g_1|^2 \le (x_1^2 + x_2^2 + x_3^2 + x_4^2)^2 \frac{a_1^2}{M^2},$$

$$|g_2|^2 \le (x_1^2 + x_2^2 + x_3^2 + x_4^2)^2 \frac{9}{4} b_3^2,$$

$$|g_3|^2 \le (x_1^2 + x_2^2 + x_3^2 + x_4^2)^2 \frac{c_1^2 c_2^2}{4}.$$

So

$$f^T f = g_1^2 + g_2^2 + g_3^2 + g_4^2 \le \left(\frac{a_1^2}{M^2} + \frac{9}{4} b_3^2 + \frac{c_1^2 c_2^2}{4} \right) \left(x_1^2 + x_2^2 + x_3^2 + x_4^2 \right)^2.$$

From assumption 1, it follows that

$$f^T f \le L \left(\frac{a_1^2}{M^2} + \frac{9}{4} b_3^2 + \frac{c_1^2 c_2^2}{4} \right) \bar{x}^T \bar{x} \le \frac{L \left(\frac{a_1^2}{M^2} + \frac{9}{4} b_3^2 + \frac{c_1^2 c_2^2}{4} \right)}{\lambda_{\min}\left(P_{i_k} \right)} \bar{x}^T P_{i_k} \bar{x}.$$

Then

$$2 f^T P_{i_k} \bar{x} \le f^T f + \bar{x}^T P_{i_k}^T P_{i_k} \bar{x} \le \left[\frac{L \left(\frac{a_1^2}{M^2} + \frac{9}{4} b_3^2 + \frac{c_1^2 c_2^2}{4} \right)}{\lambda_{\min}\left(P_{i_k} \right)} + \lambda_{\max}\left(P_{i_k}^{-1} P_{i_k}^T P_{i_k} \right) \right] \bar{x}^T P_{i_k} \bar{x}.$$

This implies that

$$\dot{V}_{i_k} = \bar{x}^T \left(A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} \right) \bar{x} + 2 f^T P_{i_k} \bar{x}$$

$$\le \left\{ \lambda_{\max}\left[P_{i_k}^{-1} \left(A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} \right) \right] + \lambda_{\max}\left(P_{i_k}^{-1} P_{i_k}^T P_{i_k} \right) + \frac{L \left(\frac{a_1^2}{M^2} + \frac{9}{4} b_3^2 + \frac{c_1^2 c_2^2}{4} \right)}{\lambda_{\min}\left(P_{i_k} \right)} \right\} V_{i_k}\left(\bar{x}(t) \right).$$

It leads to

$$\dot{V}_{i_k}(\bar{x}(t)) \le b_k V_{i_k}\left(\bar{x}(t) \right), \qquad t \in [t_{k-1}, t_k),$$

which implies that

$$V_{i_k}\left(\bar{x}(t) \right) \le V_{i_k}\left(\bar{x}(t_{k-1}) \right) \exp\left[b_k (t - t_{k-1}) \right],\ t \in [t_{k-1}, t_k). \qquad (9)$$

Substituting (10) into (11) leads to

$$\lambda_{\min}\left(P_{i_k} \right) \bar{x}^T(t) \bar{x}(t) \le V_{i_k}\left(\bar{x}(t) \right)$$
$$\le \lambda_{\max}\left(P_{i_k} \right) \bar{x}^T(t_{k-1}) \bar{x}(t_{k-1}) \exp\left[b_k (t - t_{k-1}) \right],$$

and

$$\bar{x}^T(t) \bar{x}(t) \le \rho \bar{x}^T(t_{k-1}) \bar{x}(t_{k-1}) \exp\left[b_k (t - t_{k-1}) \right],\quad t \in [t_{k-1}, t_k),$$

On the other hand, from (5), it follows that,

$$\bar{x}^T(t) \bar{x}(t) = \bar{x}\left(t_k^- \right)^T \left(I + B_{2k} \right)^T \left(I + B_{2k} \right) \bar{x}(t_k)$$
$$\le \lambda_{\max}\left[\left(I + B_{2k} \right)^T \left(I + B_{2k} \right) \right] \bar{x}^T\left(t_k^- \right) \bar{x}\left(t_k^- \right)$$
$$\le \beta_k \bar{x}^T\left(t_k^- \right) \bar{x}\left(t_k^- \right),$$

where $\beta_k \ge 0,\ \ k = 1, 2, \ldots$

For $t \in [t_0, t_1)$,

$$\bar{x}^T(t) \bar{x}(t) \le \rho \bar{x}^T(t_0) \bar{x}(t_0) \exp\left[b_1 (t - t_0) \right],$$

which leads to

$$\bar{x}^T\left(t_1^- \right) \bar{x}\left(t_1^- \right) \le \rho \bar{x}^T(t_0) \bar{x}(t_0) \exp\left[b_1 (t_1 - t_0) \right],$$

and

$$\bar{x}^T(t_1) \bar{x}(t_1) \le \beta_1 \bar{x}^T\left(t_1^- \right) \bar{x}\left(t_1^- \right) \le \rho \beta_1 \bar{x}^T(t_0) \bar{x}(t_0) \exp\left[b_1 (t_1 - t_0) \right].$$

Similarly, for $t \in [t_1, t_2)$,

$$\bar{x}^T(t) \bar{x}(t) \le \rho \bar{x}^T(t_1) \bar{x}(t_1) \exp\left[b_2 (t - t_1) \right]$$
$$\le \rho^2 \beta_1 \bar{x}^T(t_0) \bar{x}(t_0) \exp\left[b_1 (t_1 - t_0) + b_2 (t - t_1) \right].$$

In general, for $t \in [t_{k-1}, t_k),\ \ k = 1, 2, \ldots$

$$\bar{x}^T(t) \bar{x}(t) \le \bar{x}^T(t_0) \bar{x}(t_0) \rho^k \beta_1 \beta_2 \cdots \beta_{k-1} \exp\left[b_1 (t_1 - t_0), \right.$$
$$\left. + b_2 (t_2 - t_1) + b_k (t - t_{k-1}) \right]. \qquad (10)$$

Noticing (6), it follows from (10) that

$$\bar{x}^T(t) \bar{x}(t) \le \bar{x}^T(t_0) \bar{x}(t_0) \rho \exp\left[\psi_0 (t_0, t) \right], \qquad t \ge t_0,$$

which implies the origin of the nonlinear impulsive and switching system (5) is exponentially stable.

Remark 1. In inequality (6), $\sum_{i=1}^{k-1} \ln(\rho \beta_i)$ is impulsive effect, $\sum_{i=1}^{k-1} b_i (t_i - t_{i-1}) + b_k (t - t_{k-1})$ is switching effect.

Remark 2. When $\lim_{t \to +\infty} \psi(t_0, t) = -\infty$, implies the origin of the nonlinear impulsive and switching system (5) is asymptotically stable.

V. NUMERICAL SIMULATIONS

Example. We prove that the equilibrium point $S_1(1.75, -1.52, 0, 2.91)$ of the system (1) is stable under the impulsive and switching control.

Let $x_1 = x - x^*,\ \ x_2 = y - y^*,\ \ x_3 = z - z^*,\ \ x_4 = u - u^*,$ then we can rewrite (1) as

$$\dot{\bar{x}} = A\bar{x} + f(\bar{x}), \qquad (11)$$

where

$$A=\begin{bmatrix} a_1(1-\frac{2x^*}{M}) & -a_2 & -a_2 & -d_3 \\ b_3(N+z^*-2x^*) & -b_1 & b_3x^*-b_2 & 0 \\ c_1c_2z^* & 0 & c_1(c_2x^*-c_3) & 0 \\ d_1 & 0 & 0 & -d_2 \end{bmatrix},$$

$f^T(\bar{x})=[-\frac{a_1}{M}x_1^2,-b_3x_1^2+b_3x_1x_3,c_1c_2x_1x_3,0]$, $(x^*,y^*,z^*,u^*)=S_1(1.75,-1.52,0,2.91)$,and the values of a_i,b_i,c_i,d_i,M,N are the same as the values in Section 2. Correspondingly, system (11) with the impulsive and switching control can be written as

$$\begin{cases} \dot{\bar{x}}=A_{i_k}\bar{x}+f(\bar{x}), & t\in[t_{k-1},t_k), \\ \Delta\bar{x}=B_{2k}\bar{x}(t_k^-), & t=t_k, \\ \bar{x}(t_0)=\bar{x}_0, & k=1,2... \end{cases} \tag{12}$$

where $A_{i_k}=A+B_{1k}$,

$$B_{1k}\equiv\begin{bmatrix} a_{11} & a_2 & a_2 & d_3 \\ -b_3(N+z^*-2x^*) & a_{22} & -(b_3x^*-b_2) & 0 \\ -c_1c_2z^* & 0 & a_{33} & 0 \\ -d_1 & 0 & 0 & a_{44} \end{bmatrix},$$

$B_{2k}\equiv diag\{-1,-2,0,0\}$,

and

$$a_{11}=-a_1\left(1-\frac{2x^*}{M}\right)-2-\frac{L}{2}\left(\frac{a_1^2}{M^2}+\frac{9}{4}b_3^2+\frac{c_1^2c_2^2}{4}\right),$$

$$a_{22}=b_1-3-\frac{L}{2}\left(\frac{a_1^2}{M^2}+\frac{9}{4}b_3^2+\frac{c_1^2c_2^2}{4}\right),$$

$$a_{33}=-c_1(c_2x^*-c_3)-4-\frac{L}{2}\left(\frac{a_1^2}{M^2}+\frac{9}{4}b_3^2+\frac{c_1^2c_2^2}{4}\right),$$

$$a_{44}=d_2-5-\frac{L}{2}\left(\frac{a_1^2}{M^2}+\frac{9}{4}b_3^2+\frac{c_1^2c_2^2}{4}\right).$$

Let $P_{i_k}=diag\{1,1,1,1\}$, which is symmetric and positive definite, it leads to $\rho=1$.

By calculating, we have $\lambda_{\max}\left[(I+B_{2k})^T(I+B_{2k})\right]\equiv 1$, then let $\beta_k\equiv 1$, it implies that $\sum_{i=1}^{k-1}\ln(\rho\beta_i)\equiv 0$. Notice that $\lambda_{\max}\left[P_{i_k}^{-1}\left(A_{i_k}^TP_{i_k}+P_{i_k}A_{i_k}\right)\right]=\lambda_{\max}(2A_{i_k})$,

$$\frac{L\left(\frac{a_1^2}{M^2}+\frac{9}{4}b_3^2+\frac{c_1^2c_2^2}{4}\right)}{\lambda_{\min}(P_{i_k})}=L\left(\frac{a_1^2}{M^2}+\frac{9}{4}b_3^2+\frac{c_1^2c_2^2}{4}\right),$$

$\lambda_{\max}\left(P_{i_k}^{-1}P_{i_k}^{\ T}P_{i_k}\right)=1$. Then $b_k\equiv -3$.

By letting $\psi(t_0,t)=-3(t-t_0)$, $\alpha=3$, we have

$$\sum_{i=1}^{k-1}\ln(\rho\beta_i)+\sum_{i=1}^{k-1}b_i(t_i-t_{i-1})+b_k(t-t_{k-1})$$
$$=\sum_{i=1}^{k-1}b_i(t_i-t_{i-1})+b_k(t-t_{k-1})=\psi(t_0,t).$$

From Theorem, the origin of the nonlinear impulsive and switching system (12) is exponentially stable, i.e. the equilibrium point $S_1(1.75,-1.52,0,2.91)$ of (1) is exponentially stable.

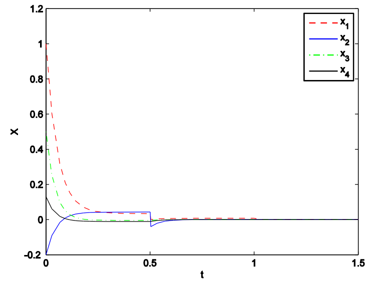

Fig2 The time evolution of the system (12).

Choosing the values of a_i,b_i,c_i,d_i,M,N are the same as the values in Section 2, the initial states of the controlled system (14) are selected as $(1,0.4,2,0.5)$, and the behaviors of the states (x_1,x_2,x_3,x_4) of the controlled chaotic system (14) with time are displayed in Fig. 2.

VI. Conclusions

In this paper, we have investigated the exponential stability of the four-dimensional energy resources supply-demand system under the impulsive and switching control. Based on the theory of impulsive, some sufficient conditions have been presented to guarantee the exponential stability. Finally, the example with its simulation has been given to demonstrate the effectiveness of the theory results.

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