A Nonlinear Stochastic Optimization Model for Water distribution network problem with reliability consideration

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***Abstract*— Water treatment and distribution is undoubtedly of high priority to ensure that communities could gain access to safe and affordable drinking water. Therefore the distribution network should be designed systematically. We propose a nonlinear stochastic optimization model for tackling this problem under the consideration of reliability in water flows. The nonlinearities arise through pressure drop equation. We adopt sampling and integer programming based approch for solving the model. A direct search algorithm is used to solve the integer part.**

***Keywords*—water network problem, nonlinear programming, neighbourhood search, active constraint method**

I. Introduction

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One fundamental issue regarding to the increase of population correlated to the increase of industrial and agriculture activities has motivated the need for a more rational use of water resources. A well planned of water resources development, their distribution, and their utilization has been put forward for research, particularly in North Sumatera Province, Indonesia. This type of plan belongs to the management of what is called Water Resources Management (WRM). Traditionally, the objectives of WRM are to preserve limited water resources and to utilize them effectively based on environmental consideration. From mathematical point of view we can cast the WRM problem as an optimization model ([15], [16], [17]). In real world situation, particularly in North Sumatera Province, the WRM problems contain a multiperiod feature. In this case the associated mathematical optimization models consist of thousand of constraints and variables depending on the level of adherence required in order to reach a significant representation of the system. Another complicated situation, these problems are typically characterized by a level of uncertainty about the value of hydrological exogenous inflows and demand patterns. On the other hand inadequate values assigned to them could invalidate the results of the study. When the statistical information on data estimation is not enough to support a stochastic model or when probabilistic rules are not available, an alternative approach could be in practice that of setting up the scenario analysis technique [2], [7], and [5]. The scenario generating approach considers a set of statistically independent scenarios, and exploits the inner structure of their temporal evolution in order to obtain a robust decision [13]. Another examples of scenario approach in WR management can found in [12] and [14].

In this paper we treated the uncertainties in demand and water flow to be in reliability pattern. Therfore, in this case, the optimization model of WRM considering reliability can be written as a stochastic constrained programming (SCP). [19] who firstly proposed SCP. This type of programming is mainly concerned with the problem in which decision maker must give a decision before the random variables are revealed. The decision may not satisfy the reliability constraints in some degree, but the probability of decision satisfying these constraints cannot be less than some given confidence level α.

Traditionally, we can solve the SCP by converting the stochastic problem into an equivalent deterministic linear programming, and then we solve the result using deterministic algorithm [18]. However, generally SCP cannot be converted into deterministic linear programming and convex programming. Under some assumption it is possible to write a mixed integer programming (MIP) model for SCP [20].In this paper we solve the MIP problem using direct search approach.

II. MODEL FORMULATION

A. For Deterministic Case

Firstly we formulate a WRM model in a deterministic framework Meaning that we have a previous knowledge of the time sequence of inflows and demand. For a wider time horizon we could extend sufficiently the analysis by We extend the analysis to a sufficiently by assuming a time step (period), *t*. In order to get a representative of the variability of hydrological inflows and water demands in the system, the scale and number of time-steps considered must be adequate. Due to the static form,we should be able to represent the physical system by a direct graph. In graph, nodes could represent sources, demands, reservoirs, water resources , hydropower station site, etc. A dynamic multiperiod network derived by replicating the basic graph for each period supports the dynamic problem. We then connect the corresponding reservoir nodes for different consecutive periods by additional arcs carrying water stored at the end of each period.

The elements considered in the mathematical formulation are not expressed in detail. However, the reduced model is still adequate to reveal the uncertainties in WRM.

It is understood that to define a general mathematical model for WRM problem quite difficult , It can be seen in the model that we take into account the elements of a system as general as possible based on the most typical characterization of a problem considered. Different elements can be considered or ignored updating constraints and objective. In this paper we describe only some of them. For detailed description of this approach can be found in [14], [8], and [6]. In the following we refer to the dynamic network G = (N, E) where N is the set of nodes and E is the set of arcs. T represents the set of time-steps t.

Nomenclature.

Set (Nodes) :

- N_r set of reservoir nodes: the set represents surface water resources along with storage capacity.
- N_d set of demand nodes: these are for civil and industrial irrigation among others.
- N_h set of hydroelectric nodes: these are for non-consumptive nodes associated with hydroelectric plants
- N_c set of confluence nodes: for example, river confluence, withdraw connections for demands satisfaction, etc.
- R set of capacitated arcs: arcs with flow that provide a cost or a benefit per unit of flow.
- TF set of arcs which transfer works in operational or in project state.

There are other sets of arcs are, such as, emergency transfers, spilling arcs, among others

Parameters for N_r

- $Y_{j\max}$ max storage volume for inter-periods transfer.
- $\rho^t_{j\max}$ ratio between max volume usable in each period t and the reservoir capacity.
- $\rho^t_{j\min}$ ratio between min stored volume in each period and reservoir capacity.
- δ_j gradient of the relationship between the reservoir surfaces and volumes.
- l_j evaporation losses per unit of reservoir surface.
- inp^t_j hydrological input to the reservoir

c_j^t spilling cost

M_j max allowed capacity

m_j min allowed capacity

γ_j construction costs;

Parameter for N_d

P_j population

d_j^t unitary demand

π_j^t request program

c_j deficit cost;

$P_{j\max}$ max population

$P_{j\min}$ min population

β_j net construction benefits

κ_j operating cost;

Parameters for N_h

H_j production capacity.

π_j^t production program .

b_j production benefit.

$H_{j\max}$ max production capacity

$H_{j\min}$ min production capacity

γ_j construction cost; required data for a confluence node *j*:

I_j^t hydrologic input (if arcs are natural streams);

Parameter for TF

F_a transfer capacity

$\rho_{a\max}^t$ ratio between max transferred volumes and capacity

$\rho_{a\min}^t$ ratio between min transferred volumes and capacity

c_a operating cost.

$F_{a\max}$ max transfer capacity

$F_{a\min}$ min transfer capacity

γ_a construction cost

Variables

We can devide the variables considered in the model as flow and project variables. Flow variables may refer to different type of water transfer such as: water-transfer in space along arc connecting different nodes at the same time, water transfer in arc connecting homologous nodes at different time and so on. Other variables which refer to the project state and they are associated to the dimension of future works: reservoirs capacities, pipes dimensions, irrigation areas, are called project variables Constraints in the model include: mass balance equations, demands for the centers of water consumption, evaporation at reservoirs, conservation of mass, and conservation of energy. Relations between flows variables and planning works, upper and lower bounds on decision variables. Therefore we can define some variables and corresponding constraints as follows.

y_j^t portion of stored water at reservoir *k* at the end of period *t* that can be used in next periods.

p_j^t water demand at civil demand center *j* in period *t*.

h_j^t water trough hydropower plant *j* (hydroelectric power station) in period *t*.

x_a^t flow on arc a.

Due to the multiperiod dynamic network structure, mass balance constraints are defined in each node $i \in N$. Moreover, lower and upper bounds constraints are defined in some arcs $a \in A$ to represent some particular limits as for transfer arcs TF.

Constraints

For each time period t, there are several constraints needed.

$$\rho^t_{k\min} Y_{j\max} \le y^t_j \le \rho^t_{k\max} Y_{j\min}$$

this constraints ensures that, in each period, the used volume of the reservoir k be in the prescribed range. In an operational state is a data while in a project state it is a decision variable. In the last case it is bounded by:

$$m_j \le Y_{j\max} \le M_j, \quad \forall j \in N_r$$

Constraints to ensure the fulfillment of the demand.

$$p^t_j \ge \pi^t_j d^t_j P_j, \quad \forall j \in N_d$$

P_j is a data while in a project state it is a decision variable. In the last case it is bounded by:

$$P_{j\min} \le P_j \le P_{j\max}, \forall j \in N_d$$

Cosntarints for flow on production capacity.

$$h^t_j \le \pi^t_j \alpha_j H_j, \quad \forall j \in N_h$$

In an operational state H_j is a data while in a project state it is a decision variable. In the last case it is bounded by:

$$H_{j\min} \le H_j \le H_{j\max}, \quad \forall j \in N_h$$

Constraint for transferred volume

$$\rho^t_{a\min} F_a \le x^t_a \le \rho^t_{a\max} F_a, \quad \forall a \in \boldsymbol{TF}$$

In this constraints the transferred volume in arc a should be in the prescribed range. In project state F_a is a decision variable. It is bounded by:

$$F_{a\min} \le F_a \le F_{a\max}, \quad \forall a \in \boldsymbol{TF}$$

Water flow for each arc in each period t should meet civil water demand. This can be written as

$$\sum_{a \in TF} x^t_a \ge d_j p^t_a, \forall a \in TF$$

It is necessarily to include conservation of mass in the network system. Normally, conservation of mass states that, for a steady system, the flow into and out of the system must be the same. This relationship must be met for the entire network and for individual nodes. A node is included in a network model at (1) a demand location and/or (2) a junction where two or more pipes combine. The mass balance equation is written for each node in the network as:

$$\sum Q_{in} - \sum Q_{out} = Q_{demand}$$

where Q_{in} and Q_{out} are the flows in pipes entering or exiting the node and Q_{demand} is civil demand at that location. These demands are uncertain since they are estimated from the local user base that cannot be predicted exactly since they vary nearly continually. In addition, the demand is typically represented as a lumped demand for users near the node.

The second important governing equation is a form of conservation of energy that describes the relationship between the energy loss and pipe flow. The most commonly used head loss equation for water networks, the Hazen-William s equation, will be the only such relationship considered in this paper. In English units, the equation is written as:

$$H_i - H_j = h_l = \frac{4.73 L Q^{4.73}}{C^{1.852} D^{4.87}}$$

where D is the pipe diameter, L is the pipe length, Q is the pipe flow, and C is the Hazen-Williams pipe roughness coefficient. h_l is the head or energy loss in the pipe. H_i and H_j are the energy at nodes at the ends of the pipe measured in dimensions of length. Using this equation, conservation of energy can be written in several ways. Most often, it is written for energy loss around a loop. The two conservation relationships can be used to develop a set of nonlinear equations that can be solved for the pipe flows, Q, and nodal heads, H.

Objective function

The objective function considers weights on variables, that is costs and benefits as well as penalties, associated to flow and project variables. Following simplified notation defined in this paper the objective function is expressed as:

$$Min \sum_{j\in N_r} \gamma_j Y_{j\max} - \sum_{j\in N_d} \beta_j P_j + \sum_{j\in N_d} \gamma_j H_j + \sum_{a\in \boldsymbol{TF}} \gamma_a F_a + \sum_{a\in \boldsymbol{R}} c_a x_a^t + \sum_{j\in N_d} \kappa_j H_j$$

B. For chance dynamic model

The presented model is called chance-model to put in evidence that we need assurance that water demand and water flow are met as expected. Therefore the model will take what is called chance constrained programming.

In the next section, we address the stochastic chance constraint programming and how to solve the problem.

III. CHANCE CONSTRAINED PROGRAMMING

Mathematically a chance-constrained optimization problem can be formulated as follows [4].

$$\min_{x\in X} f(x) \text{ subject to } \Pr\{W(x,\xi)\le 0\}\ge 1-\varepsilon \quad (1)$$

where $X\subset R^n$ is defined as a deterministic feasible region, $f:R^n\to R$ represents the objective to be minimized, ξ is a random vector whose probability distribution is supported on set $\Xi\subset R^n$, $W:R^n\times R^d\to R^m$ is a constraint mapping, 0 is an m-dimensional vector of zeroes, and $\varepsilon\in(0,1)$ is a confidence parameter. Problem (1) seeks a decision vector x from the feasible set X that minimizes the function $f(x)$ while satisfying the chance constraint $W(x,\xi)\le 0$ with probability at least $1-\varepsilon$. The probability distribution of ξ is assumed to be known.

From the model of water distribution network mentioned earlier, there are two constraints can be written as probabilistic constraint, such as:

$$\Pr\left(p_j^l \le \pi_j^l d_j P_j\right)\ge 1-\varepsilon$$

$$\Pr\left(h_j^l \le \pi_j^l \alpha_j H_j\right)\ge 1-\beta$$

In this paper we consider an approximation of the chance constraint problem (1) where the true distribution of ξ is replaced by an empirical distribution corresponding to a Monte Carlo sample. In this case a sample average approximation problem can be used for solving the problem. The sampled approximation problem is a chance-constrained problem with a discrete distribution and can be quite difficult. We address an integer programming based approaches for solving it.

IV. SAMPLE AVERAGE APPROXIMATION

For simplicity, we assume, that the constraint function $W: R^n\times R^d\to R$ in (1) is scalar valued. Some constraints $W_i(x, \xi)\le 0$, $i = 1, \dots, m$, can be equivalently replaced by one constraint $W(x, \xi) := \max_{1\le i\le m} W_i(x, \xi)\le 0$. The chance-constrained stochastic program (1) can be rewritten as

$$\min_{x\in X} f(x) \text{ subject to } q(x)\le\varepsilon \quad (2)$$

where $q(x) := \Pr\{W(x, \xi) > 0\}$

Now let $\xi^1, \dots \xi^N$ be an independent identically distributed (iid) sample of N realizations of random vector ξ. Given $x\in X$ define

$$\hat{q}_N(x) := N^{-1}\sum_{j=1}^{N} 1_{(0,\infty)}(W(x,\xi^j)),$$

where $1_{(0,\infty)}: R\to R$ is the indicator function of $(0, \infty)$. That is, $\hat{q}_N(x)$ is equal to the proportion of realizations with $W(x,\xi^j) > 0$ in the sample. For some given $\gamma\in(0, 1)$ consider the following optimization problem associated with a sample $\xi^1, \dots, \xi^N$,

$$\min_{x\in X} f(x) \text{ subject to } \hat{q}_N(x)\le\gamma \quad (3)$$

we can say that the problems (2) and (3) as the true and sampled average approximate (SAA) problems, respectively, at the respective risk levels ε and γ.

From the model, it is assumed that X is compact, $f(.)$ is continuous, $W(x, .)$ is measurable function for every $x\in R^n$, and $W(\cdot,\xi)$ is continuous for almost every ξ. Then the functions $q(x)$ and $\hat{q}_N(x)$ are lower-semicontinuous, and the true problem (1) and the SAA problem (3) are guaranteed to have optimal solutions if they are feasible. Let $X^*(\varepsilon)$ and $\hat{X}_N(\gamma)$ denote the set of optimal solutions of the true and SAA problems, respectively, $v(\varepsilon)$ and

$\hat{v}_N(\gamma)$ denote the optimal value of the true and SAA problems, respectively.

III. SOLVING SAMPLE APPROXIMATIONS

As mentioned in the previous Section that we can generate as well as validate candidate solutions to the chance constrained problem (2) by solving (several) sampled approximations. In this section we discuss approaches for solving these problems.

If $\gamma = 0$, from Eq. (3) the SAA problem reduces to

$$\min_{x\in X} f(x) \text{ subject to } W(x,\xi^j) \le 0, \quad j=1,\dots,N \quad (4)$$

When the functions $f(\cdot)$ and $W(\cdot, \xi^j)$ for $j = 1, \dots, N$ are convex (linear) and the set X is convex (polyhedral) then (4) is a convex (linear) program, and can be solved using the usual method. We can then consider increasing the risk level γ in the SAA problem. However with $\gamma > 0$ the SAA problem is a chance constrained optimization problem (with a finite distribution) and is NP-hard even in very simple settings [18]. In this paper we consider an integer programming based approach.

The SAA problem (3) can be formulated as the following mixed-integer problem (MIP)

$$\begin{aligned} &\min \quad f(x) \\ &\text{Subject to} \\ &\quad W(x,\xi^j) \le M_j z_j \quad j=1,\dots,N \\ &\quad \sum_{j=1}^{N} z_j \le \gamma N \\ &\quad z_j \in \{0,1\} \\ &\quad x \in X \end{aligned} \quad (5)$$

where z_j is a binary variables and M_j is a large positive number such that $M_j \ge \max_{x\in X} W(x, \xi^j)$ for all $j = 1, \dots, N$. Note that if z_j is 0 then the constraint $W(x, \xi^j) \le 0$ corresponding to the realization j in the sample is enforced. On the other hand $z_j = 1$ does not pose any restriction on $W(x, \xi^j)$. The cardinality constraint $\Sigma_{j=1}^{N} z_j \le \gamma N$ requires that at least N of the N constraints $W(x, \xi^j) \le 0$ for $j = 1, \dots, N$ are enforced.

Even in a linear setting (i.e., the functions f and W are linear in x and the set X is polyhedral) moderate sized instances of the MIP (10) are typically very difficult to solve. The difficulty is due to the fact that the continuous relaxation of (5) (obtained by dropping the integrality restriction on the z variables) provides a weak relaxation, and hence slows down the branch-and-bound algorithm. This difficulty can be alleviated by strengthening the formulation (5) by addition of valid inequalities or reformulation. Such improved formulations have tighter continuous relaxation gaps and can serve to significantly cut down solve times.

A variety of approaches for strengthening special classes of the MIP (5) have been proposed. Here we discuss an approach for the case of joint probabilistic constraints where the uncertain parameters only appear on the right-hand side, i.e.,

$$W(x,\xi) = \max_{i=1,\dots,m} \{\xi_i - W_i(x)\}$$

Note that the facility sizing example (2) is of this form. By appropriately translating, we assume that $\xi_j^i \ge 0$ for all i and j. The MIP (5) can then be written as

$$\begin{aligned} &\min \quad f(x) \\ &\text{subject to} \\ &\quad W_i(x) \ge v_i \quad i=1,\dots,m \\ &\quad v_i + \xi_i^j z_j \ge \xi_i^j \quad i=1,\dots,m, \quad j=1,\dots,N \\ &\quad \Sigma_{j=1}^{N} z_j \le \gamma N \\ &\quad z_j \in \{0,1\} \quad j=1,\dots,N \\ &\quad x \in X, \ v_i \ge 0 \quad i=1,\dots,m \end{aligned} \quad (6)$$

Note that we have introduced the auxiliary variables v_i for $i = 1, \dots, m$ to conveniently represent $W_i(x)$. As before, if z_j is 0 then the constraints $W_i(x) \ge \xi_i^j$ for $i = 1, \dots, m$ corresponding to the realization j in the sample is enforced.

V.THE ALGORITHM

Let $x = \lfloor x \rfloor + fr$, $0 \le fr \le 1$ be the (continuous) solution of the relaxed problem, $\lfloor x \rfloor$ is the integer component of non-integer variable x and fr is the fractional component.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\left\{ f_{i^*}, 1 - f_{i^*} \right\}$

Step 2. Do a pricing operation $v_{i^*}^T = e_{i^*}^T B^{-1}$.

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T a_j$ with j corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\sigma_{ij}} \right| \right\}$$

I. *For nonbasic j at lower bound*

If $\sigma_{ij} < 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\left(1 - \delta_{i^*}\right)}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\left(1 - \delta_{i^*}\right)}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

II. *For nonbasic j at upper bound*

If $\sigma_{ij} < 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\left(1 - \delta_{i^*}\right)}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\left(1 - \delta_{i^*}\right)}{\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased form LB or decreased from UB. If none go to next i^*.

Step 4. Calculate $\sigma_{j^*} = B^{-1} a_{j^*}$ i.e. solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}.

Step 5. Ratio test; there would be three possibilities for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

If j^* lower bound

Let $A = \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\}$,

$B = \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\}$, $C = \Delta$

The maximum movement of j^* depends on: $\theta^* = \min\left(A, B, C\right)$

If j^* upper bound

Let $A' = \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\}$,

$B' = \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\}$, $C' = \Delta$

The maximum movement of j* depends on: $\theta^* = \min\left(A', B', C'\right)$

Step 6. Exchanging basis for the three possibilities

1. If A or A'
   - $x_{B_{i'}}$ becomes nonbasic at lower bound $l_{i'}$
   - x_{j^*} becomes basic (replaces $x_{B_{i'}}$)
   - x_{i^*} stays basic (non-integer)

2. If B or B'
   - $x_{B_{i'}}$ becomes nonbasic at upper bound $u_{i'}$
   - x_{j^*} becomes basic (replaces $x_{B_{i'}}$)
   - x_{i^*} stays basic (non-integer)

3. If C or C'
   - x_{j^*} becomes basic (replaces x_{i^*})
   - x_{i^*} becomes superbasic at integer-valued

Repeat from step 1.
Stop if there are no infeasible basic variable to be processed.

VI. CONCLUSIONS

This paper presents a chance constrained programming model for water resources management planning. We use sample average approximation to transform the stochastic model to become a deterministic mixed integer programming. Then we use a direct search approach for solving the result model.

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