Reconstruction and adjustment of surfaces from a 3-D point cloud

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Abstract *— In this work, we present a 3-D reconstruction method based on the use of Bezier surfaces, combined with three-dimensional smoothing. The main idea is to generate, from a point cloud from scanning a 3-D object, a representation of surfaces from which we proceed to the 3-D reconstruction of the object in question. Then, we present the results of our method applied to some objects.*

Keywords *—3-D, reconstruction, Bezier surfaces, smoothing, point cloud.*

I. Introduction

Currently, the 3-D image is the perfect medium to convey information in the most complete way. This three-dimensional representation allows a better understanding of analytical problems, control by vision, reconstruction of different types of structures: anatomical, geological for topographical applications, scanning of objects of art ... etc.

Most representatives of real scenes images are wholly or partially composed of curves or surfaces. The raised issue is to generate an optimal representation of surfaces, according to specific criteria, from which the reconstruction will be carried out.

Several methods have been developed and used on literature [1], [2], [16], [17], and the success of these methods depends largely on the complexity of the considered objects as well as the intended application. Our approach is to develop a surface adjustment algorithm from a 3D point cloud. This algorithm covers a broad spectrum of applications and can be applied to objects of more or less complex structures.

For the treatment, we used data that was derived from the technique of active triangulation [3], [11], [12], [18] as well as those retrieved from synthetic images. This type of database respects the rectangular topology which facilitates processing steps. That is to say that "every non limit point is connected to its four neighbours" (Fig.1).

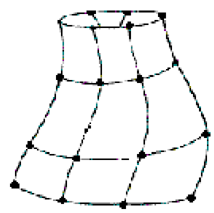


Fig.1. Example of control points

II. Approach:

It begins with a step of initialization of starting control points. These points represent the peaks of the squares. These, linked together form a status of mosaic form. Each square generates a suitable surface using Bernstein-Bezier representation. Thus, for each square we calculate the Euclidean distance between the points of the database which are inside the considered square and the surface generated. This criterion is based upon a comparison between the Euclidean distance of the center of the square and the one of the surface, and a predefined threshold, leads us to either the acceptance of the square, or an adaptive subdivision phase. This phase consists of dividing the square into four "sub-squares" creating new control points and new squares and so on. Ultimately, with a null threshold acceptance, we will end up with all the points of the initial database.

III. General Construction method of Bernstein-Bezier surfaces

A. *Representation of Bernstein-Bezier*

This representation is based upon control points forming a grid; each square of the grid generates adequate surface using Bezier representation (eq 1).

Bezier surfaces are generated from squares specified by their sixteen control points housed in a matrix [P] (Fig.2).

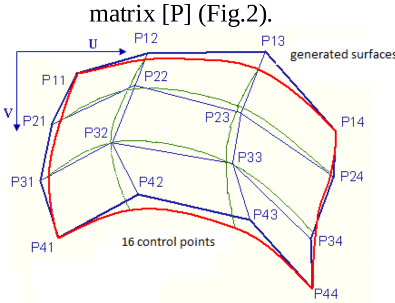


Fig.2 Area Q generated by (eq 1)

$$Q\,(u,v) = \left[u^3 u^2 u\ 1\right]\left[B\acute{e}z\right]\left[P\right]\left[B\acute{e}z\right]^t\left[v^3 v^2 v\ 1\right]^t$$

(eq 1)

With : $0 \le u \le 1$ and $0 \le v \le 1$

$$[B\acute{e}z] = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } [P] = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}$$

The elements of the matrix [P] are the coordinates (x, y, z) represented in a work object, for each control point.

Therefore, the generated surface will be expressed in the same coordinate system [4], [5], [6], [7] and [13].

B. The continuity constraint

Each square surface must be enclosed with its neighbours ensuring the continuity and a smooth aspect. This continuity is formulated by constraint equations. The binding curve (Fig.3) is determined by replacing v by 0 in equation (1) we obtain then:

$$Q(u,0) = \left[u^3 u^2 u\ 1\right]\left[B\acute{e}z\right]\left[P_{11} P_{12} P_{13} P_{14}\right]$$

(eq 2)

With $0 \le u \le 1$

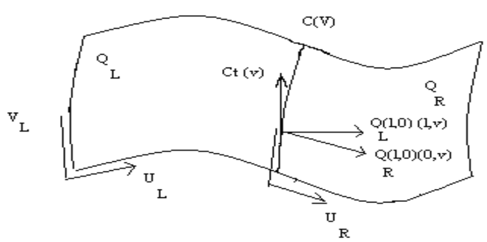


Fig.3 binding curve

We will use a continuity of type G1 defined by:

Given two adjacent surfaces (Fig.3)

Q_L (u_L, v_L) and Q_R (u_R, v_R).

We have then:

C (v) = Q_L (u_L=1, v_L=v) = Q_R (u_R=0, v_R=v)

C (v) is the binding curve between the two surfaces.

The two surfaces Q_L and Q_R bind with a G1 [] continuity, if the tangent plane, composed of two vectors $Q^{(1,0)}$ (u, v) and $Q^{(0,1)}$ (u, v) is continuous. Which give:

$$Det[C^{(1)}(v), Q_L^{(1,0)}(1,v), Q_R^{(1,0)}(0,v)] = 0 \qquad 0 \le v \le 1$$

(eq 2)

With:

$$Q_L^{(0,1)} = \frac{\partial Q_L(1,v)}{\partial v} \quad Q_R^{(1,0)} = \frac{\partial Q_R(0,v)}{\partial u}$$

To simplify the approach, we will use the following constraint:

$$Q_L^{(1,0)}(1,v) = \beta 1_u Q_R^{(1,0)}(0,v) \qquad 0 \le v \le 1$$

(eq 3)

The two vectors are collinear for each v between 0 and 1.

By applying (eq 1) to (eq 3), we find:

$$\begin{bmatrix}-3 & 3 & 0 & 0\end{bmatrix}\left[P^L\right]\left[B\acute{e}z\right]\begin{bmatrix}v^3 & v^2 & v & 1\end{bmatrix}^t = \beta l_u \begin{bmatrix}0 & 0 & -3 & -3\end{bmatrix}\left[P^R\right]\left[B\acute{e}z\right]\begin{bmatrix}v^3 & v^2 & v & 1\end{bmatrix}^t$$

(eq4)

For $0 \le v \le 1$ which give:

$$\begin{bmatrix}-1 & 1 & 0 & 0\end{bmatrix}\left[P^L\right] = \beta l_u \begin{bmatrix}0 & 0 & -1 & 1\end{bmatrix}\left[P^R\right]$$

(eq 5)

This expression corresponds to the following set of equations:

$$P_{2j}^L - P_{1j}^L = \beta l_u \left(P_{4j}^R - P_{3j}^R\right) \qquad j=1 \text{ to } 4$$

(eq 6)

Applying the same analysis to the common curve C (u), we find:

$$P_{i2}^T - P_{i1}^T = \beta l_v \left(P_{i4}^B - P_{i3}^B\right) \quad i=1 \text{ to } 4$$

(eq 7)

With $[P^B]$ and $[P^T]$ which represent the matrices of control points of the two adjacent surfaces B and T binded by the curve C (u).

This sequence of constraints can be regrouped into a system of six equations (eq1 to eq6), which allows the localization of the eight points surrounding the common point of 4 surfaces.

To facilitate the positioning of the control points, we will rename them according to their position relative to the point Oij which represents the intersection of the four adjacent surfaces $Q_{i,j}$, $Q_{i,j+1}$, Q_{i+1j}, $Q_{i+1,j+1}$. (Fig.4).

C. 3.3 Positioning of control points

From point Oij taken in the Pij database, we determine its 8 neighbours noted N, S, E, W, NE, NW, SE and SW (Fig.4).

The control points N, S, E, W, connected to O_{ij} by equations (9-2) and (9-5) are connected by lines passing through the center Oij. These points must be in the same plane which is the plane tangent to the considered object point O_{ij}

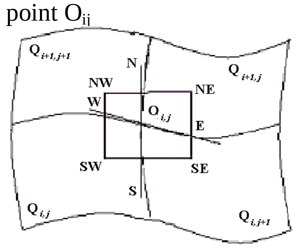


Fig.4.

Let $z = ax + by + c$ the equation of the plane tangent to the object point O_{ij}. The estimation of this equation is done by minimizing the criterion:

$$J = \sum_{i=1}^{n} (z_i - ax_i - by_i - c) \qquad \text{(eq 8)}$$

The parameters ($\beta 1_u$) and ($\beta 1_v$) must be identical along the direction u (v).

Equations (6) and (7) become:

$NE - N = \beta 1_i (N - NW)$ (eq 9-1)
$E - O_{ij} = \beta 1_i (O_{ij} - W)$ (eq 9-2)
$SE - S = \beta 1_i (S - SW)$ (eq 9-3)
$NW - W = \beta 1_j (W - NS)$ (eq 9-4)
$N - O_{ij} = \beta 1_j (O_{ij} - S)$ (eq 9-5)
$NE - E = \beta 1j (E - SE)$ (eq 9-6)

These points are shown in Figure.4.

IV. 4. ADAPTIVE SUBDIVISION

A. Initialization phase :

The initial grid is chosen with Nu × Nv squares (patchs) in the direction of u and v. This choice must check the following criteria:

- The squares must be adjacent surfaces and well distributed throughout the object
- For each square, we determine the matrix [P] of control points using the method described in section 3.2.

B. Adaptive Subdivisions Phase

Several works, among them: [6], [14] and [15], deal with the subdivision. The method described in [6], uses an adaptive approach. It is summed up in two stages: Creation of new control points by subdivision and positioning of these.

1. Creation of new control points.

Let a square Qij with its 8 square neighbours (Fig.5)

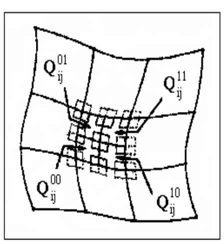


Fig.5

A square is divided into four sub-squares. These, themselves, are bi-cubic surfaces of Bernstein-Bezier. Their control matrix $\boldsymbol{P}^{ij}$ is calculated on the basis of the control matrix of the initial square.

Adjustment of New control points

For each new point deducted in the first step, we search for its correspondent (the closest) in the database. Then, we determine its eight neighbouring points N, S, E, W, NE, NW, SE and SW using equations (9) and following the method described in section 3.2

This subdivision process is repeated until the acceptance of the square according to a specific criterion. The applied criterion consists of calculating the Euclidean distance between the center of a square and ~~of~~ the one of the generated surface, and compares it to a predefined threshold.

2. Algorithm

Begin
Initialization surfaces
Selection Control Points
Test d: Euclidean distance between each point of the database and the generated area
If d < ε then squared accepted
generated area
else
square of the division into sub square
Positioning and new points
End if
End

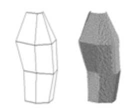
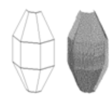
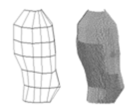
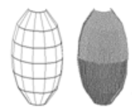
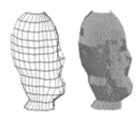
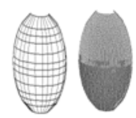
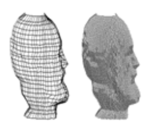
3. Results AND DISCUSSION:

Let two point clouds from two databases.

1- A synthetic sphere of 1953 points 3D (x, y, z) corresponding to 63 horizontal points of same z and 31 points with the same positioning angle.

2- A statuette of Victor Hugo analyzed with Cartesian coordinates of 8280 points (x, y, z) corresponding to 120 horizontal points of same z and 69 points of the same positioning angle.

The initial base is selected with Nu = 3 and Nv = 6, which corresponds to 18 squares.

	relative results at first database	relative results at second database
(a)		
(b)		
(c)		
(d)		
	Fig.6: statue of Victor Hugo	**Fig.7: Synthetic sphere**

a : Initialization Phase

b, c, d: successive subdivisions.

Figures 6 and 7 illustrate the successive iterations of the adaptive subdivision.

On the left is the processed subdivision patchs (grids) and on the right surfaces generated from these grids.

The table below shows the results for a threshold equal 2

TABLE I

ITERATIONS

Database Victor Hugo	New Points	Accumulation Points
Initialisation	24	24
1st Subdivision	60	84
2nd Subdivision	221	305
3rd Subdivision	823	1128

N.B: The number of points in the initial database 1 is 8280.

We observe that from the third subdivision, the generated surfaces resemble reality.

Conclusion

We presented in this paper a new reconstruction method 3-D, based on the use of Bezier surfaces, from a point cloud. Applied to 2 types of images, a synthetic and real, this method satisfactory results both visually and in terms of processing complexity. An improvement of this operation may be performed by developing techniques for automatic selection of control points in the initial mesh and that of the threshold used for the calculation of the Euclidean distance.

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